67. ON PROPAGATION OF ELECTRO-MAGNETIC WAVES THROUGH THE ATMOSPHERE

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§1. Introduction

The subject of propagation of electromagnetic waves through the atmosphere is mostly treated in terms of the ray method. The present state of our knowledge is summarized in two excellent reports: (1) Report on the present state of our knowledge of the Ionosphere by S. K. Mitra published in the *Proceedings of the National Institute of Sciences of India*, Vol. I, No. 3, December 1935; hereafter referred to as Report 1; (2) Radio Exploration of the Upper Atmospheric Ionisation by E. V. Appleton in the *Reports on the Progress of Physics*, Vol. II, 1936; hereafter referred to as Report 2. Prof. Appleton remarks on page 133 of Report 2:

'The discussion in the present section has so far been written in terms of a ray treatment. But it is well known that it is not possible adequately to describe optical phenomena in terms of geometrical optics when the refractive index μ varies appreciably within a wavelength in the medium in a direction normal to the wavelength. Now for total reflection at normal incidence, the wavelength becomes infinitely large as $\mu \to 0$, so that in this case the ray treatment needs further justification.'

The wave treatment of the problem has been tried by Hartree[1] in three important papers. In the first paper, he obtained, in the absence of the magnetic field, the differential equations for wave propagation and calculated the electric and magnetic fields. In the second, he derived an expression for μ, taking the earth's magnetic field into account. In the third paper, he obtained approximate expressions for the optical and equivalent paths for a stratified medium in which μ varies slowly. Reviewing the work of Hartree, Appleton says [Report 2, p 133], 'On the whole, however, it may be said that the work of Hartree shows that the errors made in using a ray treatment are not serious in most practical problems.'

It is, however, not difficult to find illustrations which show the complete insufficiency of the ray treatment even for cases ordinarily met with: as an illustration, let us take the condition of reflection. A wave propagated vertically upwards is supposed to be reflected from the spot where the refractive index μ reaches the value 0, on account of increase in electron or ion concentration.

As Appleton has shown (see Report 1, p. 142, figs. 3-8), where q is plotted against electron-concentration, this criterion gives us the conditions of reflection of the *o*-wave ($p^2=p_o^2$), as well as of the *x*-wave ($p_o^2=p^2 \pm pp_h$). But μ can be zero, only if the collision frequency can be neglected. In general, however, the collision frequency is never zero, and the refractive index is complex, of the form $q=\mu+i\chi$, where χ depends on the collision frequency and the ion-concentration and is not zero at any height excepting the lowest, so that q can never take a negative value even if the real part $\mu=0$. For example, see the q-curves drawn by M. Taylor[2] and Gobau[3]. They have shown that q takes up a steady, small, positive value with increasing values of ion-concentration, due to finite value of χ. It may be mentioned, however, that according to the magneto-ionic theory, both μ and χ are functions of ion-concentration, and of the collision frequency ν, each of which again is a function of the height z. None of the curves drawn either by Taylor or Gobau represents the actual variation of q with z. For this purpose, the best course would be to make use of a three-dimensional representation with electron-concentration $\left(p_o^2=\frac{4\pi Ne^2}{m}\right)$ along the x-axis, ν=collision frequency along the y-axis and q along the z-axis; p^2 and ν are to be given all possible values, we get a surface representing q, and the actual (q, z) curve for the atmosphere is obtained by taking a section of this surface through points which represent the actual values of ν, and p_o^2 at a point z in the atmosphere. This has not yet been attempted, but it can be shown that q can actually never become zero. In the F-layer, there is probably some justification for neglecting collision as $\nu/p \simeq 10^{-3}$, but this cannot be said of the E-layer where $\nu/p \simeq 1$.

On account of these difficulties, several workers have tried to formulate other criteria for the reflection of waves. Booker[4] says that appreciable reflection of an incident wave can take place only from such stratum, where either the variation of q per unit wavelength is considerable or $q \neq 0$. One of us[5] has shown that a better criterion for reflection is that group-velocity should be zero. This criterion can, however, in the present state of our knowledge be used only when collisions can be neglected, for Rayleigh's theorem that group velocity $u=\frac{\delta \nu}{\delta k}$, where ν=frequency, k=wave number, holds only when the waves constituting a group are undamped. We are not aware of any work which treats satisfactorily the case when the component waves are damped. However, neglecting collisions, one of us, Rai[5], found that the criterion group-velocity $\mu=0$ for reflections gives us the following four conditions for reflection:—

$$\left.\begin{array}{lll} (\alpha) & p_o^2=p^2-pp_h \ldots\ldots\ldots\ldots\ldots & x\text{-wave} \\ (\beta) & p_o^2=p^2.\dfrac{p^2-p^2_L}{p^2-p^2_h} \ldots\ldots\ldots\ldots & x\text{-wave} \\ (\lambda) & p_o^2=p^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots & o\text{-wave} \\ (\delta) & p_o^2=p^2+pp_h \ldots\ldots\ldots\ldots\ldots & x\text{-wave} \end{array}\right\} \quad \ldots (1)$$

Condition (β) is new and is not given by the hitherto assumed condition for reflection, i.e. $\mu=0$. The actual existence of reflection (β) was detected at Allahabad by Pant and Bajpai[6], and, in fact, gave occasion for a revision of our ideas regarding the criterion for reflection.

The cases (α) to (δ) have been further discussed in a paper by Bajpai and K. B. Mathur[7]. For the sake of illustration, we may work out from their paper the electron concentration required for the above four modes of reflection, taking p=23·3 kilocycles, $f=\frac{23\cdot 3}{2\pi}=3\cdot 70$,

we find that

$$\left.\begin{array}{llll} \text{Reflection } (\alpha) \text{ takes place when} & N= & \cdot 88\times 10^5/\text{cm}^3 \\ \quad (\beta) \quad ,, \quad ,, \quad ,, & N= & 1\cdot 40\times 10^5/\text{cm}^3 \\ \quad (\gamma) \quad ,, \quad ,, \quad ,, & N= & 1\cdot 52\times 10^5/\text{cm}^3 \\ \quad (\delta) \quad ,, \quad ,, \quad ,, & N= & 2\cdot 40\times 10^5/\text{cm}^3 \end{array}\right\} \quad . \ (2)$$

The question now arises how the same x-wave can get reflected from three different strata at one and the same time. If a wave is propagated vertically upwards, it gets split up into an o-wave, and an x-wave, which are propagated with different velocities in the ionosphere. On reaching the level where $N=\cdot 88\times 10^5/\text{cm}^3$, the x-wave gets reflected. *But is this reflection complete or partial* ? According to the ray treatment this should be complete, as the vanishing of group-velocity means that there is no further forward propagation of energy by the waves. But Pant and Bajpai[6] in this laboratory noticed the reflection of the x-wave according to the method (β), and Toshniwal[8] observed a threefold splitting of the wave, presumably of the x-wave, one of which he interpreted as mode (β). This observation was later verified by Leiv Harang[9] and recently all the four conditions of reflection have been verified by R. Jouaust and his co-workers. These cases show that reflection according to (α) is incomplete, even when we get the requisite electron-concentration, and the x-wave can sometimes leak through the layer, and get reflected under conditions which are still to be investigated, from the higher regions (β) and (δ). These cases therefore call for a revision of the treatment which has so far been pursued. But these are not the only cases which call for a revision of the ray treatment. Another is the existence of simultaneous reflection from layers at widely different heights. The best known example of this type is afforded by the case of the so-called sporadic E-layer reflection, also called Abnormal Region E-ionization by Appleton (for a detailed account, see Report 2, p. 159, where a reference to original papers will be found). It was first discovered in 1930 that 'echo-reflection often occurs from an atmospheric level approximately to that of region E for electric wave frequencies which are higher than the critical frequency for the normal region E'. Appleton concluded that 'either the recombination of ions is prevented or there is some ionizing agent present which can influence the dark side of the earth'.

The sporadic E-layer ionization has since been investigated by a large number of workers—Schafer and Goodal, Ranzi, Ratcliffe and White, Appleton and Naismith[10] —with the object of finding out the conditions which give rise to the formation of the layer. It was first supposed that either thunderstorms or magnetic storms may give rise to a thin but concentrated layer of electrons at the E-height, but Kirby and Judson[11] have shown that the phenomenon cannot be due to extra-ionization produced by thunderstorms either occurring locally or within a radius of 300 Km., or due to magnetic storms. Berkner and Wells[12] have also confirmed this finding, and have further shown that sporadic E-reflection appears to be increasing in frequency as we proceed from the magnetic equator to the poles. This has been confirmed by L. Harang.[13] Apparently, the sporadic layer is formed by some focussing action of the earth's magnetic field on the electrons at the E-layer. But we are not concerned in this paper with the causes giving rise to the sporadic E-layer but with the phenomenon that the same wave can get reflected from this layer, which is at the height of 100 Km., as well as from the much higher F-layer at one and the same instant (of course neglecting the time of transit).

The apparent explanation seems to be that a layer of electrical particles of abnormal density but extreme thinness is formed at the height of the normal E-layer. This layer reflects partly the energy of the waves, but part of the energy of the incident waves leaks through the layer, and gets reflected from the upper F_2-layer. This phenomenon cannot be understood if $\mu=0$ is taken as the condition of reflection, for then reflection from the lower layer would be complete.

The M-reflections. Another phenomenon which points to the same conclusion is the existence of M-reflections investigated by Ratcliffe and White[14] and by Zenneck and Gobau. In this case the wave penetrates the E-layer, gets reflected from the F-layer, but the returned wave, instead of proceeding straight to the ground, gets reflected from the top of the E-layer, again gets reflected from the F-layer, and then proceeds straight to the ground. This phenomenon and the related E and F reflections show that the same wave can partly be reflected and can also leak through the same layer.

Let us see how all these phenomena can be explained.

§ 2.

For carrying out the programme sketched in §1, we have to start from the original Maxwellian equations, and derive equations for the propagation of electric and magnetic vectors associated with the electro-magnetic wave in the atmosphere, taking the effect of ion-concentration, collision and the earth's magnetic field into consideration. This has been done in a separate paper; here only the results which will be necessary for the present discussion are quoted: We consider only vertical propagation (axis of z). Let (E_x, E_y), (H_x, H_y) denote the components of the electric and magnetic vectors associated with the signal. We further take the magnetic meridian as our plane of xz. Then it can be shown that the wave equations for their propagation are given by

$$\left.\begin{aligned} \frac{dE_x}{dz} &= -\frac{1}{c}\frac{dH_y}{dt}, \quad \frac{dE_y}{dz} = \frac{1}{c}\cdot\frac{dH_x}{dt} \\ \frac{dH_x}{dz} &= \frac{iL}{c}\cdot\frac{dE_x}{dt} + \frac{K_2}{c}\cdot\frac{dE_y}{dt} \\ \frac{dH_y}{dz} &= -\frac{K_1}{c}\frac{dE_x}{dt} - \frac{iL}{c}\cdot\frac{dE_y}{dt} \end{aligned}\right\}, \quad \ldots (3)$$

where K_1, K_2 are complex dielectric constants. L may be called mutual dielectric constant. Further, these quantities satisfy the equations

$$\left.\begin{aligned} \frac{d^2E_x}{dz^2} &= \frac{K_1}{c^2}\cdot\frac{d^2E_x}{dt^2} - \frac{iL}{c^2}\cdot\frac{d^2E_y}{dt^2} \\ \frac{d^2E_y}{dz^2} &= \frac{iL}{c}\cdot\frac{d^2E_x}{dt^2} + \frac{K_2}{c^2}\cdot\frac{d^2E_y}{dt^2} \\ \frac{d^2H_x}{dz^2} &= \frac{K_2}{c^2}\cdot\frac{d^2H_x}{dt^2} - \frac{iL}{c^2}\cdot\frac{d^2H_y}{dt^2} \\ \frac{d^2H_y}{dz^2} &= \frac{iL}{c^2}\cdot\frac{d^2H_x}{dt^2} + \frac{K_1}{c^2}\cdot\frac{d^2H_y}{dt^2} \end{aligned}\right\}. \quad \ldots (4)$$

We can write out values of K_1, K_2, L in the general case by using the following notations, which as far as possible conform to those used by Appleton and Ratcliffe, and Mitra in their respective reports.

h ....Earth's magnetic field, components h_x, h_y, h_z.

p_h ....Larmor Frequency... $\frac{eh}{mc}$.

p_x, p_y, p_z ....Components of Larmor Frequency.

p ....Pulsatance of the E.M. Wave.

$p_0^2 = \frac{4\pi Ne^2}{m}$ ....Where N is the number of electrons or ions per c.c. e, m are their charge and mass.

ν..........Collision frequency for electrons or ions.

$$(\omega_x, \omega_y, \omega_z) = \frac{1}{p}(p_x, p_y, p_z), \ \omega = \text{resultant of } (\omega_x, \omega_y, \omega_z)$$

$$= \frac{p_h}{p} = \frac{eh}{mcp}.$$

$$r = \frac{p_0^2}{p^2} = \frac{4\pi e^2}{mp^2}\cdot N.$$

$$\beta = 1 - \frac{i\nu}{p} = 1 - i\delta, \ \delta = \frac{\nu}{p}.$$

$$q = \text{complex refractive index} = \mu + i\chi.$$

We have in general when damping is not neglected

$$\left.\begin{aligned} K_1 &= 1 - r\cdot\frac{\beta^2 - r\beta - \omega_x^2}{C'}, \quad K_2 = 1 - r\cdot\frac{\beta^2 - r\beta}{C'} \\ C' &= \beta(\beta^2 - \omega^2) - r(\beta^2 - \omega_z^2), \quad L = \frac{-r(\beta - r)\,\omega_z}{C'} \end{aligned}\right\}. \quad \ldots (5)$$

The general solution of equations (3) and (4) is very much complicated and is not attempted here. We take the simplified case when collisions can be neglected, *i.e.* $\beta = 1$. In this case we have

$$\left.\begin{aligned} K_1 &= 1 - r\cdot\frac{1 - r - \omega_x^2}{(1-\omega^2) - r(1-\omega_z^2)}, \\ K_2 &= 1 - r\frac{1-r}{(1-\omega^2) - r(1-\omega_z^2)}, \\ L &= -\frac{r(1-r)\,\omega_z}{(1-\omega^2) - r(1-\omega_z^2)}. \end{aligned}\right\} \quad \ldots (6)$$

For the solutions of equations (3) and (4), we may first utilize the facts that every quantity E or H is proportional to exp. (ipt)—then bearing in mind that E_x, E_y are now only functions of z, and introducing a new variable $u = \frac{zp}{c}$, we have

$$\left.\begin{aligned} \frac{dE_x}{du} &= -iH_y; & \frac{dE_y}{du} &= iH_x \\ \frac{dH_x}{du} &= -LE_x + iK_2E_y; & \frac{dH_y}{du} &= -iK_1E_x - LE_y \\ \frac{d^2E_x}{du^2} &= -K_1E_x + iLE_y; & \frac{d^2E_y}{du^2} &= -iLE_x - K_2E_y \\ \frac{d^2H_x}{du^2} &= -K_2E_x - iLE_y; & \frac{d^2H_y}{du^2} &= iLH_x - K_1H_y \end{aligned}\right\}. \quad (7)$$

[The last two equations in H_x, H_y are strictly not quite rigorous].

In the solution of these equations, we may as a first step regard the quantities K and L as constants, *i.e.*, not varying with z, and then we get all the results deduced by

Appleton and others regarding splitting of the waves into o and x-components, their polarization, refractive indices, and the conditions for reflection [conditions (α), (γ), (δ)]. When the expressions (5) are utilized, the usual expressions for refractive index, and absorption coefficient of the two components can also be deduced.

But a little reflection shows that this procedure is not strictly justified, for K and L, both involve r and ν (ionization and collision frequency) and these vary with height. We are therefore, in general, not justified in proceeding with the solution on the assumption that K and L are constants.

To illustrate this point, let us take the simplest possible case, *viz.*, that of propagation in the magnetic equator. We have then $\omega_z=0$, $\omega_x=\omega$. We have therefore

$$K_1 = 1 - r,\ K_2 = 1 - \frac{r(1-r)}{(1-r)-\omega^2},\ L=0 \qquad \dots (8)$$

The equations reduce to:

$$\left.\begin{aligned} &o\text{-wave: } \frac{d^2E_x}{du^2}+(1-r)E_x=0;\ \frac{d^2H_y}{du^2}+(1-r)H_y=0 \\ &x\text{-wave: } \frac{d^2E_y}{du^2}+\left\{1-\frac{r(1-r)}{(1-r)-\omega^2}\right\}E_y=0 \\ &\qquad \frac{d^2H_x}{du^2}+\left\{1-\frac{r(1-r)}{(1-r)-\omega^2}\right\}H_x=0 \end{aligned}\right\} \quad (9)$$

For the o-wave, it is usual to take $1-\frac{p_0^2}{p^2}$ as equal to μ^2, and the vanishing of this gives us the condition for reflection of the o-wave: *viz.* $p_0^2=p^2$; similarly we may take $\mu_x^2=1-\frac{r(1-r)}{(1-r)-\omega_x^2}$ and this vanishes when $p_0^2=p^2\pm pp_h$.

But since r is a function of z, the procedure is not justified. We cannot, in fact, talk of a refractive index in the usual sense. These equations are of the form

$$\frac{d^2\phi}{du^2}+k^2\phi=0 \qquad \dots \qquad \dots \qquad \dots (10)$$

where k^2 is not a constant, but a function of (u).

Equations of this type were first discussed by Lord Rayleigh[15]; see 'On the propagation of waves through a stratified medium with special reference to the question of reflection.' The following treatment is based on Lord Rayleigh's with the necessary modification. The same treatment was later given by Gans[16], apparently without a knowledge of Rayleigh's previous work. It may also be mentioned that Gamow[17] has used the same method in his famous work on the 'Penetration of the Potential Barrier of Nuclei of Atoms by high energy particles.'

Let us put $\phi=e^{s(u)}$. We have then

$$\frac{d\phi}{du}=\phi\frac{ds}{du},\ \frac{d^2\phi}{du^2}=\left\{\frac{d^2s}{du^2}+\left(\frac{ds}{du}\right)^2\right\}\phi. \qquad \dots (11)$$

The differential equation (10) takes the form

$$\frac{d^2s}{du^2}+\left(\frac{ds}{du}\right)^2+k^2=0\ . \qquad \dots \qquad \dots (12)$$

If it is possible to neglect the first term with respect to the second, *i.e.* if

$$\left|\frac{\frac{d^2s}{du^2}}{\left(\frac{ds}{du}\right)^2}\right| = \left|\frac{d}{du}\cdot\frac{1}{\frac{ds}{du}}\right| \ll 1,\ \text{we have}$$

$$\left(\frac{ds_1}{du}\right)^2+k^2=0\ . \qquad \dots \qquad \dots (13)$$

Here s_1 denotes the first approximation to s. We have then

$$\frac{ds_1}{du}=\pm ik,\ \text{or}\ \pm l,$$

$$\text{and}\ \phi=C\exp.\left[\pm i\int_{u_1}^{u_2}k\,du\right],\ \text{or}\ C\exp.\left[\pm\int_{u_1}^{u_2}l\,du\right]. (14)$$

The second form holds when k^2 is negative$=-l^2$.

Let us now work out a second approximation. We now regard C as varying with u or z. It can be easily shown that C satisfies the equation

$$\frac{d^2C}{du^2}+2ik\frac{dC}{du}+ik'C=0, \qquad \dots \qquad \dots (15)$$

where $k'=\frac{dk}{du}$.

If we suppose that $\frac{d^2C}{du^2}$ can be neglected, we easily obtain from (15)

$$C=\frac{A}{\sqrt{k}}\ \text{or}\ \frac{A'}{\sqrt{l}},$$

according as k^2 is positive or negative.

So the second approximation gives us, as the solution of (10)

$$\phi=\frac{A}{\sqrt{k}}e^{\pm i\int k ds}, \qquad \dots \qquad \dots (16A)$$

$$\text{or}\quad \frac{A'}{\sqrt{l}}\cdot e^{\pm\int l ds}. \qquad \dots \qquad \dots (16B)$$

This solution can be regarded as correct if

$$\frac{d^2}{dz^2}(k^{-\frac{1}{2}})=0\ .\ \dots \qquad \dots \qquad \dots (17)$$

We shall assume that this condition is satisfied, though actually, as can easily be seen this is strictly not the case even approximately. Let us now see how these solutions can be applied to the present case. As the wave moves up in the atmosphere, $r=\frac{4\pi e^2}{mp^2}\cdot\mathcal{N}$ gradually approaches the value 1, and ultimately may exceed 1, so that $k^2=1-r$ becomes negative. The actual state of affairs is represented in Fig. 1. Here the abscissa 'Z' represents height, the ordinate $r=\frac{4\pi e^2}{mp^2}\cdot\mathcal{N}$ is proportional to the electron-concentration. The distribution of electrons is represented by the curve OPQR which is of course fancied. If wave of some other frequency were employed, the position of the line $r=1$(A, A_1, A_2, A_3) would shift with respect to the curve, *e.g.* for $p_1 > p$ the line would go up, and ultimately may cross over the crest of the curve OPQR. For $p_2 < p$ the line AA_1A_2 would move down as shown in Fig 1.

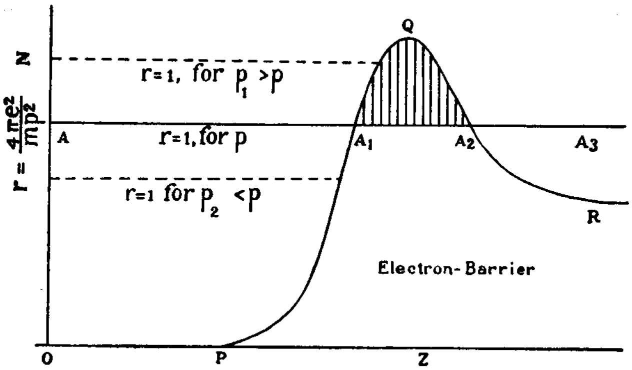

Fig. 1

From A to A_1, the value of $k^2 = 1 - r$ is positive, at A_1 it becomes zero, between A_1 and A_2 it is negative, and then it is again positive. We have to write different suitable solutions of (10) for the three different regions, and apply the conditions of continuity $\left(\text{equality of } \phi \text{ and } \frac{d\phi}{du}\right)$ at the transition points A_1 and A_2. Then it can be easily shown that there is a forward and backward wave of type (16A) in the region between O and A_1, in the region A_1A_2, the disturbance is damped (16B) and there are two components, beyond A_2 there is only one wave (16A) with the negative sign. The coefficient of transmission through the barrier is given by

$$T = 4 \left(\frac{1-r_1}{1-r_2}\right)^{\frac{1}{2}} e^{-\int_{u_1'}^{u_2'} \sqrt{1-r}\cdot du}, \quad .. \ (18)$$

where r_1 is the limiting value of r in the region (OA_1), r_2 in the region A_3. u_1, u_2 are the co-ordinates for the points A_1, A_2 respectively. We have, restoring the old co-ordinates

$$T = 4\left(\frac{p^2}{p^2 - p_{03}^2}\right)^{\frac{1}{2}} \exp.\left[-\frac{1}{c}\int \sqrt{p_0^2 - p^2}\cdot dz\right]_{z_1}^{z_2}. \ (19)$$

We shall try to calculate its value in some representative cases. In the equation (19), $p_0^2 = \frac{4\pi e^2}{m} \cdot \mathcal{N}$, where $\mathcal{N}$ is the actual electron-concentration at a height z within A_1A_2. As 'p' is fixed, we can put $\frac{4\pi e^2}{mp^2} = \mathcal{N}_1$, so $\mathcal{N}$ is the electron-concentration at the layer where reflection takes place. Hence the index of e in (19) is given in the case of electrons by

$$-\frac{1}{c}\left(\sqrt{\frac{4\pi e^2}{m}}\right) \int_{z_1}^{z_2} \sqrt{\mathcal{N}_0 - \mathcal{N}_1}\cdot dz$$

$$= -1{\cdot}9 \times 10^{-6}. \int_{z_1}^{z_2} \sqrt{\mathcal{N} - \mathcal{N}_1}\, dz, \quad .. \ (20)$$

and m refers to electrons. If they refer to ions, we have

$$\text{the index} = -8{\cdot}3 \times 10^{-9} \int_{z_1}^{z_2} \sqrt{\mathcal{N} - \mathcal{N}_1}\, dz. \quad .. \ (21)$$

The value of the integral cannot be obtained unless we know the form of the electron-concentration curve above the line $r = 1$. Let us suppose that it is in the form of an isosceles triangle, so that for the rising half,

$$\mathcal{N} = \mathcal{N}_1 + \alpha(z - z_1),$$

then the value of $\mathcal{N}$ at the peak will be $\mathcal{N}_m$. Then the half-breadth of the barrier $= \frac{\mathcal{N}_m - \mathcal{N}_1}{\alpha}$. For the descending half, $\mathcal{N} = \mathcal{N}_m - \alpha z$. We must have some idea of the values of $\mathcal{N}_1$, $\mathcal{N}_2$, α and the half-breadth in order to be able to make some calculations.

It is well known, however, that for the *E*-layer, α is large, and the thickness of the layer is small, while for the *F*-layer, α is small, and the thickness of the layer is large. But as figures are available only for the *F*-layer, we shall give a calculation based on data for *F* alone.

In a case cited by Appleton, the values of $\mathcal{N}$ are given as follows:—

Penetration Frequency for
(h = 300 Km.) = 4 megacycles, $\mathcal{N} = 1{\cdot}98 \times 10^5$
(h = 210 Km.) = 3·7 ,, $\mathcal{N} = 1{\cdot}70 \times 10^5$

So we have $\frac{d\mathcal{N}}{dz} = \frac{2{\cdot}8 \times 10^4}{90 \times 10^5} = 3 \times 10^{-3}$ electrons per cm.

$$= -1{\cdot}90 \times 10^{-6} \alpha^{\frac{1}{2}} b^{\frac{3}{2}} \frac{4}{3},$$

where b = half breadth of the barrier.

Using these values we have the index

$$= -1{\cdot}90 \times 10^{-6} \alpha^{\frac{1}{2}} b^{\frac{3}{2}} . \frac{4}{3},$$

where b = half-breadth of the barrier.

Taking into account the factor $\left(\frac{p^2}{p^2 - p_0^2}\right)^{\frac{1}{2}}$, we find that the amount of energy transmitted falls to half of its value, for a barrier of the form shown in fig. 1, and for gradient and electron density characteristic of the *F*-layer, if the half-breadth of the layer is about 1·5 Km. This calculation is of course fancied. but it shows that the wave can penetrate some thicknesses of the ionized layer without appreciable diminution in intensity. For the *E*-layer, α is larger, but b is smaller, hence if the thickness of the layer is of the order of a kilometer a part of the energy of the incident wave may be transmitted, though the value of μ is zero for the wave transmitted at the point where it meets the electron-barrier.

This treatment of penetration of electron or ion-barriers by e.m. waves cannot be utilized for the x-wave, which has singularities at

(α) $p_0^2 = p^2 - pp_h$, (β) $p_0^2 = p^2 . \frac{p^2 - p_L^2}{p^2 - p_h^2}$, and

(δ) $p_0^2 = p^2 + pp_h$.

The problem is still under consideration.

In conclusion, we wish to express our thanks to Dr. G. R. Toshniwal for many useful discussions and to Mr. K. B. Mathur and Dr. Rakshit for much help in the calculations.

SUMMARY

In this paper, the ray treatment of passage of e.m. waves through the ionosphere has been critically reviewed, and a wave treatment has been given for the *o*-wave for propagation in the magnetic equator. It has been shown that contrary to the implicit assumption in the ray treatment which requires complete reflection at the point in the ion-barrier where μ falls to zero, there may be considerable penetration by the wave of the barrier, even when the thickness of the barrier amounts to several kilometres.

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68. ON THE ACTION OF ULTRA-VIOLET SUNLIGHT UPON THE UPPER ATMOSPHERE

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1—INTRODUCTION

The ordinary solar spectrum extends, as is well known, to about λ2913, the more ultra-violet parts being cut off by ozone absorption in the upper atmosphere. We have thus no direct knowledge of the distribution of intensity in the solar spectrum beyond λ2913, as it will appear to an observer situated outside the atmosphere of the earth. But it is now recognized that a number of physical phenomena is directly caused by the photochemical action of this part of sunlight on the constituents of the upper atmosphere. Such phenomena are (1) the luminous spectrum of the night sky and of the sunlit aurora,[1] (2) the ionization in the E, F and other layers which is now being intensely studied by radio-researchers all over the world[2], (3) the formation and equilibrium of ozone (see Ladenburg 1935), (4) magnetic storms and generally the electrical state of the atmosphere.

Formerly it was a debatable point whether some of these phenomena were not to be ascribed to the action of streams of charged particles emanating from the sun. There seems to be no doubt that the polar aurora and certain classes of magnetic storms are to be ascribed to the bombardment of molecules of N_2 and O_2 by such charged particles, for these phenomena show a period which is identical with the eleven year period of the sun, and are found in greater abundance, the nearer we approach the magnetic poles.[3] But there now exists no doubt that the ionization observed by means of radio-methods in the E and F_1 regions, their variation throughout day and night, and at different seasons is due to the action of ultra-violet sunlight. This was decisively proved by observations during several total solar eclipses since 1932 (Appleton and Chapman 1935). The luminous night-sky spectrum, though it has certain points of similarity to the polar aurora, is on the whole widely different, and is found on nights free from electrical disturbances. The prevailing opinion is that it is mainly due to the ultra-violet solar rays, i.e. in the course of the day sunlight is stored up by absorption by the molecules in the upper atmosphere, and again given up during the night, in one or several steps, as a fluorescence spectrum. According to S. Chapman (1930) the formation of the ozone layer and its equilibrium under different seasonal conditions is

[1] For a general account of the spectrum of the luminous night sky, see Dejardin (1936).

[2] For general information regarding investigation on the ionosphere, see Mitra and others (1936) and Appleton (1936).

[3] See for general information article by Störmer (1931). The frequency of aurorae appears to reach a maximum 20° from the magnetic pole.