WONDERS OF SPACE-TIME

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SCIENCE IS THE SEARCH FOR

UNITY IN

OUR UNIVERSE

FULL OF DIVERSITIES

UNIFYING THE WHOLE MATERIAL WORLD

NEWTON'S LAWS OF MOTION :

FORCE = MASS X ACCELERATION

F = ma

TRUE FOR ANY MATERIAL BODY OF MASS m

WE NOW LOOK INTO

NEWTONIAN MECHANICS CRITICALLY

SPACE AND TIME

ACCELERATION 'a' IS GIVEN BY

WHERE x, y, z DESCRIBES A 3-DIMENSIONAL SPACE

AND t IS TIME

dx = SPACE INTERVAL ; dt = TIME INTERVAL

QUESTION:

WHAT ARE THE CHARACTERS OF

SPACE AND TIME ?

SPACE

ACCORDING TO NEWTON :

1. TIME IS ABSOLUTE, UNIVERSAL, UNIQUE FOR

ALL EVENTS IN NATURE

2. SPACE IS ABSOLUTE, INFINITE,

3-DIMENSIONAL WHICH PASSIVELY CONTAINS

ALL THE EVENTS IN THE UNIVERSE

GEOMETRY

NEWTON ASSUMED THAT THE GEOMETRY OF

THIS SPACE IS "EUCLIDEAN"

SHORTEST DISTANCE BETWEEN TWO POINTS – STRAIGHT LINE

(REMEMBER NEWTON'S FIRST LAW OF MOTION)

EINSTEIN

• EINSTEIN QUESTIONED : NEWTONIAN VIEWPOINT ABOUT SPACE AND TIME



TIME IS NOT UNIVERSAL !

THERE IS NO UNIVERSAL CONCEPT OF TIME.

JUST AS EVERY EVENT HAS THEIR OWN POSITION IN SPACE

EACH OF THEM HAS THEIR OWN TIME !!

TIME IS LIKE A COORDINATE

JUST AS POSITION IN SPACE IS RELATIVE,

FLOW OF TIME IS ALSO RELATIVE !!

THE FACT THAT

1. ALL INERTIAL FRAMES ARE EQUIVALENT

2. SPEED OF LIGHT IS INDEPENDENT OF THE

MOTION OF THE OBSERVER LED TO NEW

CONCEPT OF TIME AND SPACE



TIME FLOWS DIFFERENTLY FOR THE OBSERVER

AND THE OBJECTS 'A' AND OBJECT 'B'. ------

FLOW OF TIME DEPENDS ON VELOCITY !!!!

 $t = t_0 / (1 - v^2/c^2)^{1/2}$

MOVING CLOCK RUNS SLOW !!!!

DISTANCE

ALSO DISTANCE BETWEEN 'A' AND 'B' DEPENDS

ON VELOCITY --

LENGTH DEPENDS ON VELOCITY

 $L = L_0 (1 - v^2/c^2)^{1/2}$

LENGTH CONTRACTS IN MOVING FRAME !!

THIS NEW CONCEPT OF RELATIVE TIME AND SPACE LED TO :

- 1. NEW CONCEPT OF MASS AND ENERGY
- 2. NEW RELATIVISTIC MECHANICS
- 3. CREATION AND DESTRUCTION OF PARTICLES

---- THIS IS 'THE SPECIAL THEORY OF RELATIVITY'

A REVOLUTIONARY CONCEPT

EINSTEIN THEN PUT FORWARD A

REVOLUTIONARY IDEA -----

THE CONNECTION BETWEEN

GEOMETRY AND FORCE !

FORCE OF GRAVITATION



EINSTEIN'S VIEWPOINT

INSTEAD OF NEWTONIAN CONCEPT OF

ACTION AT A DISTANCE

EINSTEIN CAME OUT WITH A BRILLIANT

IDEA

FLAT EUCLIDEAN SPACE



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CURVED SPACE



CLOSED

sum > 180



FLAT sum = 180

\wedge	
\square	
F	

OPEN

sum < 180

NEWTONIAN IDEA

- THE SPACE-TIME IS FLAT
- A BODY TRAVELS ALONG THE SHORTEST PATH
- i.e. STRAIGHT LINE IN ABSENCE OF FORCE
- AND ACCELERATES WHEN THERE IS FORCE
- (NEWTON'S FIRST LAW OF MOTION)

NEWTONIAN PICTURE

NO FORCE :

STRAIGHT LINE



IN PRESENCE OF FORCE

ACCELERATION



EINSTEIN'S PICTURE



FORCE AND GEOMETRY

- PRESENCE OF MASS (ENERGY) CURVES THE SPACE-TIME
- THE SHORTEST DISTANCE IS NOW A CURVED LINE GIVING
- THE ILLUSION OF ACCELERATION AND HENCE THE
- EXISTENCE OF GRAVITATIONAL FORCE
- ILLUSION OF FORCE ORIGINATES FROM
 - **CURVED GEOMETRY**

FLAT GEOMETRY

IN FLAT EUCLIDEAN LIKE GEOMETRY

THERE ALWAYS EXISTS A COORDINATE IN

WHICH DISTANCE BETWEEN TWO

POINTS (x,y,z) AND (x + dx, y + dy, z + dz) IS

 $D^2 = dx^2 + dy^2 + dz^2$

CURVED GEOMETRY

IN CURVED SPACETIME THE DISTANCE

 $D^{2} = (1 + f_{1} (x,y,z)) dx^{2} + (1 + f_{2} (x,y,z)) dy^{2} + (1 + f_{3} (x,y,z)) dz^{2}$

WHERE THE COEFFFICIENTS ARE NEVER (1,1,1) LIKE FLAT GEOMETRY

THESE COEFFICIENTS ARE CALLED

THE METRIC OF THE SPACE AND

DETERMINE WHETHER THE SPACE

IS FLAT OR CURVED

AS TIME IS ALSO LIKE A COORDINATE

THEREFORE DISTANCE IN FOUR DIMENSIONAL CURVED GEOMETRY (c = 1) $D^2 = (1 + f_1) dx^2 + (1 + f_2) dy^2 + (1 + f_3) dz^2 - (1 + f_4) dt^2$

METRIC ---- \rightarrow g_{µα} DIFFERENT COEFFICIENTS

FROM THIS METRIC :

WHICH DEPEND ON x, y, z, t THE QUANTITY $R_{\alpha\beta}(f_1, f_2, f_3, f_4)$, R CONSTRUCTED WHICH MEASURES THE CURVATURE OF THE SPACE-TIME $R = 0 IF f_1, f_2, f_3, f_4 = 0 IN SOME$ COORDINATE

EINSTEIN'S EQUATION

$R_{\alpha\beta} - g_{\alpha\beta} R/2 = K T_{\alpha\beta}$ GEOMETRY AND MATTER ARE CONNECTED

MASS (ENERGY) CURVES SPACE-TIME

PARTICLE TRAVELS IN A CURVED LINE AND WE

INMAGINE THAT THERE IS A FORCE CALLED GRAVITY

----- CALLED THE GENERAL THEORY OF RELATIVITY

TESTS OF EINSTEIN'S PICTURE

SOLVE EINSTEIN'S EQUATION TO FIND THE GEOMETRY IN

POLAR COORDINATE FOR SOLAR MASS 'M'

 $D^2 = (1 - 2GM/r)^{-1} dr^2 - (1 - 2GM/r) dt^2$

+ $r^2 d\theta^2$ + $r^2 sin^2\theta d\Phi^2$

M = 0, SPACE TIME IS FLAT

FIND THE EQUATION OF SHORTEST PATH (CALLED GEODESIC)

FOR A PLANET

THE RESULT IS AN ELLIPSE !!

SAME AS KEPLAR'S LAW OF PLANETARY MOTION AS EXPLAINED BY NEWTON'S LAW OF GRAVITATION

SOMETHING MORE !!

EINSTEIN'S THEORY PREDICTS PERIHELION PRECESSION OF PLANETARY ORBITS



COMPLETE AGREEMENT

FROM EINSTEIN'S THEORY THE ANGLE

OF ROTATION FOR THE PLANET MERCURY

= .011 DEGREE PER CENTURY

EXACTLY SAME FROM EXPERIMENTAL DATA

LIGHT BENDING EXPERIMENTS

BENDING OF LIGHT FROM DISTANT ASTROPHYSICAL OBJECTS BY SPACETIME CURVATURE ---

EXACTLEY EXPLAINED BY EINSTEIN'S THEORY



BLACK HOLES

FROM EINSTEIN'S EQUATION

WHEN A STAR COLLAPSES DUE TO IT'S OWN

GRAVITATION IT BECOMES VERY DENSE

SPACETIME AROUND IT BECOMES EXTREMELY CURVED

NOT EVEN LIGHT CAN ESCAPE !!

RECALL FOR A MASS M THE GEOMETRY $D^2 = (1 - 2GM/r)^{-1} dr^2 - (1 - 2GM/r) dt^2$ + $r^2 d\theta^2$ + $r^2 sin^2\theta D\phi^2$ FOR r < 2GM, SIGN OF COEFFICIENT OF r AND t IS REVERSED --- r CAN CHANGE LIKE TIME ONLY ALONG ONE DIRECTION !!
ONCE INSIDE r = 2GM,

ANYTHING (EVEN LIGHT) CAN

TRAVEL ONLY IN ONE DIRECTION !!

r = 2GM IS CALLED HORIZON OF

BLACK HOLE

HIGH CURVATURE LEADS TO BLACK HOLE



WHAT HAPPENS NEAR HORIZON?

AT HORIZON r = 2M, THE METRIC COEFFICIENT

BECOMES SINGULAR

CONSIDER A POINT VERY CLOSE TO HORIZON (NEAR HORIZON)

 $r = 2M + x^2/8M$ (x IS VERY VERY SMALL)

AS x ----- \rightarrow 0, r ----- \rightarrow 2M i.e. HORIZON

NEGLECTING HIGHER POWERS OF x, WE HAVE, $D^2 = -(kx)^2 dt^2 + dx^2 + (d\theta^2 + sin^2\theta d\Phi^2)/4k^2$ (k = 1/4 M) NEAR HORIZON 2-DIMENSIONAL SPACE-TIME

 $D^2 = -(kx)^2 dt^2 + dx^2$

NOTE : det(g) = 0 AT x = 0 i.e. THE SINGULARITY ON THE HORIZON PERSISTS (UNDESIRABLE FOR CALCULATION) EUCLIDEANIZE THE TIME $t \rightarrow it = \theta$

 $D^2 = x^2 d(k\theta)^2 + dx^2$

NO SINGULARITY ----- FLAT SPACE WITH RADIAL

CORDINATE x AND ANGULAR COORDINATE k0

WITH PERIOD 2π PERIOD OF EUCLIDEANIZED TIME θ IS $2\pi/k$

CONSIDER A QFT NEAR THE HORIZON

GENERATING FUNCTIONAL

 $Z \sim \sum \exp(i/h \int d^4 x L)$ $L = (\partial_t \phi)^2 - (\partial_i \phi)^2 - V(\phi)$

INTEGRATING OVER EUCLIDEAN TIME it = θ

 $Z \sim \sum \exp(-2\pi/hk H) \rightarrow PARTITION FUNCTION$

TEMPERATURE T = $hk/2\pi K_B$ ------> HAWKING TEMP.

AS k = 1/4 M,

HAWKING TEMPERATURE DECREASES WITH MASS OF THE BLACK HOLE

NEAR HORIZON THE SPACE-TIME GEOMETRY OF

A BLACK HOLE HAS

A THERMODYNAMIC EQUIVALENCE !!

THE ENTROPY OF THE BLACK HOLE IS DEFINED AS,

S = A/4

A = AREA OF THE HORIZON

WE FIND THESE QUANTITIES HAVE SIMILAR RELATIONS AMONG THEMSELVES AS LAWS OF THERMODYNAMICS INFORMATION LOSS? QUESTION : WHAT ARE THE MICROSTATES ASSOCIATED WITH A BLACK HOLE ? (STRING THEORY, LOOP QUANTUM GRAVITY)

FORCES OF NATURE

FOUR FORCES :

1.ELECTRO-MAGNETIC FORCE

- 2. WEAK NUCLEAR FORCE
- 3. STRONG NUCLEAR FORCE
- 4. GRAVITATIONAL FORCE (ORIGINATES FORM SPACETIME GEOMETRY)

- FORCES ARE CHARACTERISED BY
- CHARGE OR COUPLING (TELLS US STRENGTH OF THE FORCE)
- RANGE
- (TELLS US THE DISTANCE OF IT'S ACTION)

OLD CONCEPT OF FORCE FORCES BETWEEN PARTICLES ARE

EXTENDED AS LINES OF FORCE

OVER THE SPACE AND

EXERTED ON EACH OTHER

OLD PICTURE



NEW CONCEPT OF FORCE

ACCORDING TO QUANTUM THEORY

FORCES BETWEEN TWO PARTICLES

ARE EXCHANGE OF SOME PARTICLES

(CALLED THE QUANTA OF FORCE)

NEW PICTURE



QUANTUM OF FORCE

1. ELECTROMAGNETIC FORCE ----- PHOTON (LONG RANGE)

2. WEAK NUCLEAR FORCE ---- W⁺, W⁻, Z (SHORT RANGE)

3. STRONG NUCLEAR FORCE ----- 8 GLUONS (SHORT RANGE)

4. GRAVITATIONAL FORCE ---- GRAVITON (LONG RANGE)

QUANTUM PROCESS

ACCORDING TO QUANTUM THEORY

THE PROBABILITY OF TRANSITION FROM ONE STATE TO ANOTHER DUE TO A FORCE IS THE RESULT OF EXCHANGE OF THE QUANTA OF THE FORCE AND IS GIVEN BY SUM OF A PERTURBATION SERIES DESCRIBED BY DIFFERENT INTERMEDIATE PROCESSES

FEYNMAN DIAGRAMME





QUANTUM GRAVITY

FOR GRAVITATION INTERACTION VARIOUS TERMS

AT VARIOUS ORDERS IN THE PERTURBATION

SERIES WITH GRAVITON LOOP

HAVE THE COEFFICIENT

 $(E^2 G)^n$ (IN UNIT h = 1, c = 1)

THIS IMPLIES THAT QUANTUM EFFECT OF GRAVITY IS RELEVANT AT ENERGY SCALE E ~ $1/G^{1/2} = 10^{19}$ GeV CALLED PLANCK ENERGY (M_P) WE ARE ONLY AT 10^3 Gev !! (LHC)

CAN IGNORE QUANTUM GRAVITY EFFECTS

MICROSCOPIC WORLD



MOVING FORWARD

CAN ELECTROMAGNETIC FORCE BE EXPLAINED THROUGH SPACETIME GEOMETRY ?

YES IF WE HAVE MORE DIMENSIONS !!

METRIC COEFFICIENT OF EXTRA DIMENSION ACTS AS ELECTROMAGNETIC VECTOR POTENTIAL

(KALUZA AND KLEIN)

 $D^2 = (1 + f_1) dx^2 + (1 + f_2) dy^2 +$ $(1 + f_3) dz^2 - (1 + f_4) dt^2$ $+ g_{pp} dp^2 + g_{px} dp dx + g_{py} dp dy$ + $g_{pz}^{g} dp dz$ + $g_{pt}^{g} dp dt$ -----A FOUR VECTOR (ELECTROMAGNETIC VECTOR POTENTIAL) g_{px} , g_{py} , g_{pz} , $g_{pt} = A_{\mu}$ AND A SCALAR $g_{pp} = \phi$

THE LAGRANGIAN OF 5-DIMENSIONAL GRAVITATIONAL

THEORY BREAKS INTO :

LAGRANGIAN OF 4-DIMENSIONAL GRAVITATIONAL THEORY

+ $\partial_{\mu} \phi \partial^{\mu} \phi$ - exp (- ϕ) $F_{\alpha\beta} F^{\alpha\beta}$

IF THERE IS EXTRA SPACE DIMENSION

SPACE TIME GEOMETRY (APART FROM THE UNWANTED SCALAR)

ELECTROMAGNETIC FORCE FROM

WE FIND BOTH GRAVITATION AND

ARE THERE EXTRA DIMENSIONS ?

ACCORDING TO STRING THEORY --

YES

STRING THEORY -- A NEW HORIZON

ELEMENTARY PARTICLES ARE NOT POINT LIKE BUT

STRING-LIKE ONE DIMENSIONAL OBJECT -- CLOSED STRING OR OPEN STRING



THE STRING VIBRATES WITH

DIFFERENT FREQUENCIES i.e. WITH

DIFFERENT ENERGIES

--- EACH SUCH MODE CORRESPONDS

TO A PARTICULAR MASS OR PARTICLE

THUS SAME STRING AT DIFFERENT MODES CORRESPONDS TO DIFFERENT PARTICLES ---- A NEW UNIFICATION !! LENGTH OF STRINGS ~ 10⁻³³ cm !!

THIS THEORY IS CONSISTENT

IN SPACE-TIME DIMENSION 10 !!!!

REMAINING SIX DIMENSIONS ARE

INVISIBLE -----

A SIX DIMENSIONAL HIDDEN WORLD !!

EXTRA DIMENSIONS ARE NATURAL

IN STRING THEORY ------

GIVES RISE TO MANY NEW IDEAS

COMPACTIFICATION AND INVISIBILITY

CONSIDER A TWO DIMENSIONALSURFACE --WITH ONE COMPACT DIMENSION





IF THE RADIUS 'R' OF THE CYLINDER BECOMES SMALL -- A ONE DIMENSIONAL LINE



SUPPOSE R IS BEYOND THE

RESOLUTION OF OUR MOST

POWERFUL MICROSCOPE --

SO INVISIBLE

KALUZA KLEIN TOWER

CONSIDER ONE DIMENSIONAL WORLD DESCRIBED

BY THE COORDINATES x, y

ENERGY OF A MASSLES RELATIVISTIC

FREE PARTICLE IS

$$E^2 = p_x^2 + p_y^2$$

(TAKE c = 1)

FREE PARTICLE WAVE FUNCTION

 $\Psi(x, y) = \exp[i(p_x x + p_y y)]$ (PUT h = 1)

COMPACTIFY COORDINATE 'y' ON A CIRCLE OF RADIUS R (CALLED THE MODULUS OF EXTRA DIMENSION)

AFTER COMPACTIFICATION PERODICITY OF $\Psi\left(\,x,\,y\,\right)$ IMPLIES

 $p_v = n/R$ (n = INTEGERS)

$$E^2 = p_x^2 + n^2 / R^2$$

(n = 0,1,2,3,)

----- ENERGY OF A MASSLESS AND A TOWER OF MASSIVE PARTICLES (KK TOWER) IN ONE DIMENSION

VALUES OF KK MASSES DEPEND ON THE CHOICE OF 'R'.
NEW THEROY OF GRAVITATION ? IF THERE ARE MORE SPACE DIMENSIONS

THAN 3 THEN

WE WILL HAVE NEW THEORY OF GRAVITATION :

 $F = G M m / r^{2+d}$

(d = NUMBER OF EXTRA DIMENSIONS)

BUT IT HAS INDIRECT INFLUENCES IN 5-DIMENSION GRAVITATIONAL FORCE : $F = G_5 M m / r^{2+1}$ IF WE CAN NOT ENTER IN 5-TH DIM $F = G_5 M m / r^2 R$ ONE r IS REPLACED BY R

WE CAN GET BACK OUR FOUR DIMENSIONAL

GRAVITATION AL THEORY IF

 $G_5 = G_4 R$

SO FOR LARGE R

'G₅' CAN BE LARGE EVEN THOUGH G₄ IS SMALL

GRAVITY BECOMES STRONGER IN HIGHER DIMENSION !!

AND PLANCK ENERGY AT 5 DIMENSION

 $M_{5P} = 1/G_5^{1/2} \longrightarrow BECOMES SMALL$

APPROPRIATE CHOICE OF 'R' CAN MAKE IT CLOSE TeV

(HOPE TO SEE QUANTUM GRAVITY EFFECTS IN LHC !!)

SO WE MAY SEE THE INFLUENCE OF EXTRA SPACE IN LHC

- 1. MODIFIES THE SCATTERING PROCESS (DUE TO KK – PARTICLES)
- 2. CREATE TINY BLACK HOLES (GRAVITY BECOMES STRONG)
- QUANTUM GRAVITY EFFECTS
 (PLANCK MASS ~ TeV)

BLACK HOLES

THE TINY BLACK HOLES SHOULD RADIATE

(DUE TO QUANTUM EFFECTS)

--- HAWKING RADIATION

LHC MAY DETECT THIS RADIATIONS

DYNAMICS OF MODULUS

NUMBER OF MODULUS INCREASES WITH NUMBER OF EXTRA DIMENSIONS

THEY CAN BE FUNCTION OF OUR 3-SPACE AND TIME,

 $R = R(x,y,z,t) \longrightarrow CALLED MODULUS FIELD$

THEIR QUANTA CAN TAKE PART IN VARIOUS SCATTERING PROCESSES

NEW DEVELOPMENTS

ACCELERATING UNIVERSE – DARK ENERGY ? (73%) (CAN IT BE EXPLAINED BY MODULI DYNAMICS ?)

UNKNOWN MATTER -- DARK MATTER ? (23 %)

KNOWN MATTER -- ELECTRONS, QUARKS ETC. (ONLY 4%)

OUR UNDERSTANDING OF THE

UNIVERSE IS GOVERNED BY THE

PROPERTIES OF

SPACE AND TIME

LABORATORY AT SKY



HUBBLE SPACE TELESCOPE



LARGE HADRON COLLIDER

LHC: The Large Hadron Collider

The protons have not yet been accelerated to their full energy.

You need to supply more energy by raising the accelerator handle...

SPS LHC Ring Ring

CMS

ALICE

Contraction of the second

ATLAS

LHCh

SPS now at 306.0 GeV (68)%...

Lift handle to accelerate the stream



OUR SEARCH TO UNVEIL

THE MYSTERIES OF SPACE AND TIME

TELLS US

THE BEAUTIFUL STORY THE UNIVERSE

THANK YOU