The Carnot Process of Economic Growth and Wealth Distribution

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Coworkers: Mario Hillebrand, Christian Denk, Thorsten Fründ, Stefan Kallerhoff

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"First and Second Laws" of Macro Economics in Differential Forms

Carnot Cycles of Economic Growth and Wealth Distribution

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“First Law”
of Macro Economics

Differential forms

Exact Differential Forms:

$$df(x,y) = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$$

with

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y}$$

Differential forms

Exact Differential Forms:

$$df(x,y) = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$$ with $$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y}$$

Example 1

$$f(x, y) = x^3 y^5$$

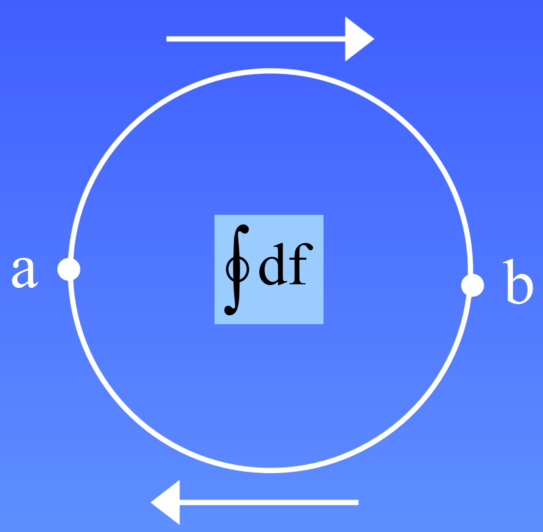
$$d f(x, y) = 3 x^2 y^5 \, dx + 5 x^3 y^4 \, dy$$

with $15 x^2 y^4 = 15 x^2 y^4$ **(exact)**

Differential forms

Exact Differential Forms:

The integral of an exact differential form d f depends on the integral limits and does not depend on the path of the integral (high school math).
The closed integral is zero.



$$\oint df = \int_a^b df + \int_b^a df = \int_a^b df - \int_a^b df = 0$$

Differential forms

Not Exact Differential Forms:

$$\delta g(x, y) = a dx + b dy$$

with

$$\frac{\partial a}{\partial y} \neq \frac{\partial b}{\partial x}$$

Differential forms

Not Exact Differential Forms:

$$\delta g(x, y) = a dx + b dy \quad \text{with} \quad \frac{\partial a}{\partial y} \neq \frac{\partial b}{\partial x}$$

Example 2

$$\delta g(x, y) = d f(x, y) / y = (3 x^2 y^5 \, dx + 5 x^3 y^4 \, dy) / y$$

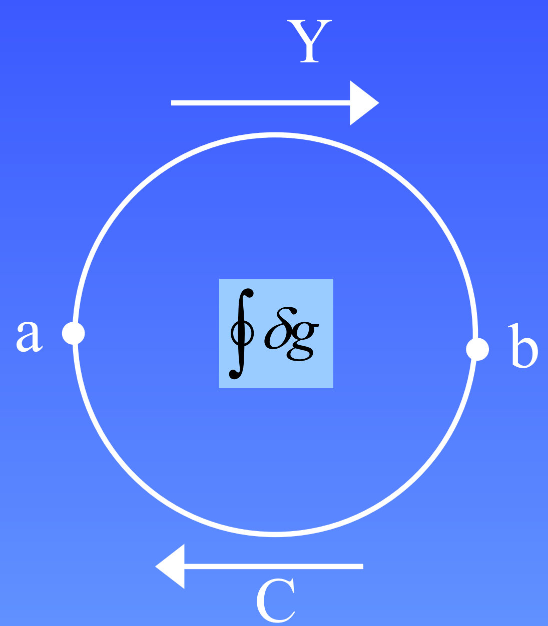
$$\delta g(x, y) = 3 x^2 y^4 \, dx + 5 x^3 y^3 \, dy$$

with $12 x^2 y^3 \neq 15 x^2 y^3$ (not exact)

Differential forms

Not Exact Differential Forms:

The integral of a not exact differential form δ g(x, y) depends on the path of the integral. The closed integral is not zero.



$$\oint \delta g(x,y) = \int_a^b \delta g_2 + \int_b^a \delta g_1 = \int_a^b \delta g_2 - \int_a^b \delta g_1 = Y - C \neq 0$$

Differential forms

Not Exact Differential Forms:

The integral of a not exact differential form $\delta g(x, y)$ depends on the path of the integral. The closed integral is not zero.

Differential forms

Not Exact Differential Forms:
The integral of a not exact differential form $\delta\, g(x, y)$ depends on the path of the integral. The closed integral is not zero.

First Law of Thermodynamics
Heat $\delta\, Q(T, p)$ is a not exact differential, the integral depends on the path (e.g. isothermal), the closed integral is not zero.

Differential forms

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First Law of Thermodynamics
Heat $\delta\, Q(T, p)$ is a not exact differential, the integral depends on the path (e.g. isothermal), the closed integral is not zero.

"First Law" of Macro Economics
Production $\delta\, q(K, L)$ is a two dim. not exact differential and depends on the specific production process. The closed cycle of production is not zero.

Differential forms

"First Law" of Macro Economics

$$\oint \delta q = \int_a^b \delta q_2 + \int_b^a \delta q_1 = \int_a^b \delta q_2 - \int_a^b \delta q_1 = Y - C = \Delta q$$

- Production leads to income (Y), consumption (C) and surplus (Δq).
- Production δ q is a not exact differential:

a) the production function q does not exist in general! or
the production function may not be calculated ex ante!

b) the integral of δq depends on the path of integration, or
the production function q depends on the production process!
Ex post, when the production process is known, q may be calculated!

“Second Law” of Macro Economics

Differential forms

Not Exact Differential Forms:

A not exact form δg may be made exact by an integrating factor y. The exact form is given by $d f = y \, \delta g$.

Example 2

$$\delta g(x, y) = d f(x, y) / y$$

Differential forms

Not Exact Differential Forms:

A not exact form δg may be made exact by an integrating factor y. The exact form is defined by $d f = y \delta g$.

Second Law of Thermodynamics

The integrating factor of the non exact form of heat δQ is the mean energy per particle or temperature, $T = E / N$.

The exact form leads to entropy defined by $d S = \delta Q / T$.

Differential forms

Not Exact Differential Forms:

A not exact form δg may be made exact by an integrating factor y. The exact form is defined by $d f = y \delta g$.

Second Law of Thermodynamics

The integrating factor of the non exact form of heat δQ is the mean energy per particle or temperature, $T = E / N$. The exact form leads to entropy defined by $d S = \delta Q / T$.

"Second Law" of Macro Economics

The integrating factor of the non exact form of production δq is a mean price level or standard of living, $T = K / N$. The exact form leads to entropy defined by $d S = \delta q / T$.

Differential forms

“Second Law” of Macro Economics:

$$dS = \delta q / T$$

- The integrating factor T is the price level of a market or the standard of living of a society (GDP per capita)
- The differential form d S is exact, or the entropy function S may be calculated ex ante!

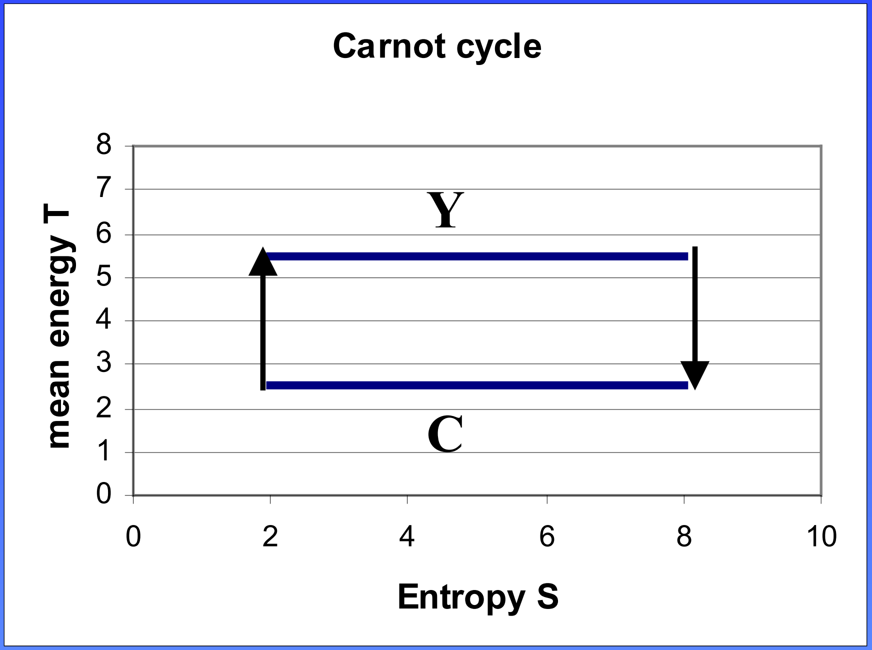
3. Entropy is related to the number of possibilities (P) of production, $S = \ln P$

Application
Production

The Carnot Process of Economic Growth and Wealth Distribution

Carnot cycle

"First and Second Law" of Economics:



$$\oint \delta q = \oint T dS = Y_T - C_T = \Delta T \cdot \Delta S$$

$$Y_T = K + T_2 \cdot \Delta S$$

$$C_T = K + T_1 \cdot \Delta S$$

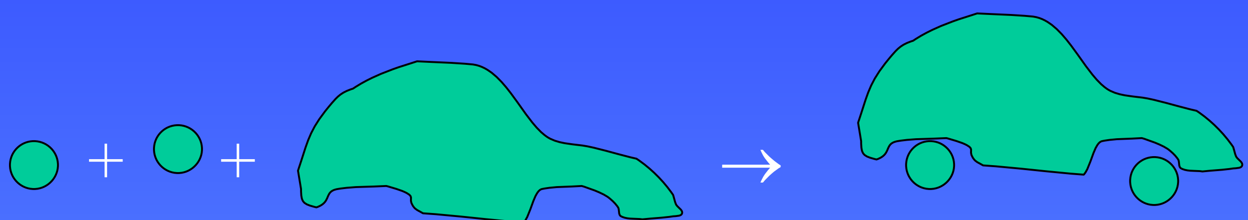
T : standard of living, price level
Y_T : income, price at const. T
C_T : costs, consumption at const. T
K : capital

The Carnot Process of Economic Growth and Wealth Distribution

Carnot : $$Y_T = K + T \cdot \Delta S$$

Production (Y) is ordering = reduction of entropy ($\Delta S = \ln P$)

Production:



Brain work: d+i+c+n+o+o+p+r+t+u → production

The Carnot Process of Economic Growth and Wealth Distribution

Carnot : $$Y_T = K + T \cdot \Delta S$$

Automobile workers:

Production Y_T is ordering (reducing the entropy) of many parts to build the car exactly according to the plans.

Automobile engineers:

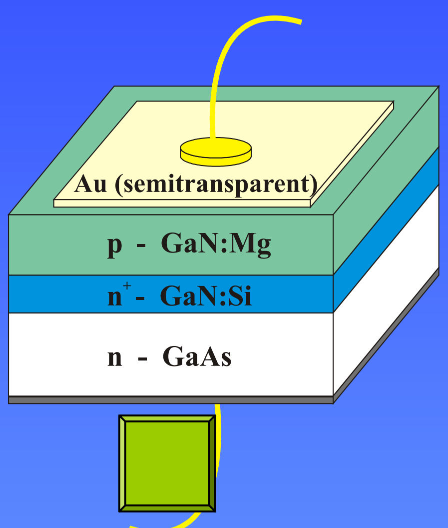
Production Y_T is ordering (reducing the entropy) of many ideas to create a new plan for a car.

The Carnot Process of Economic Growth and Wealth Distribution

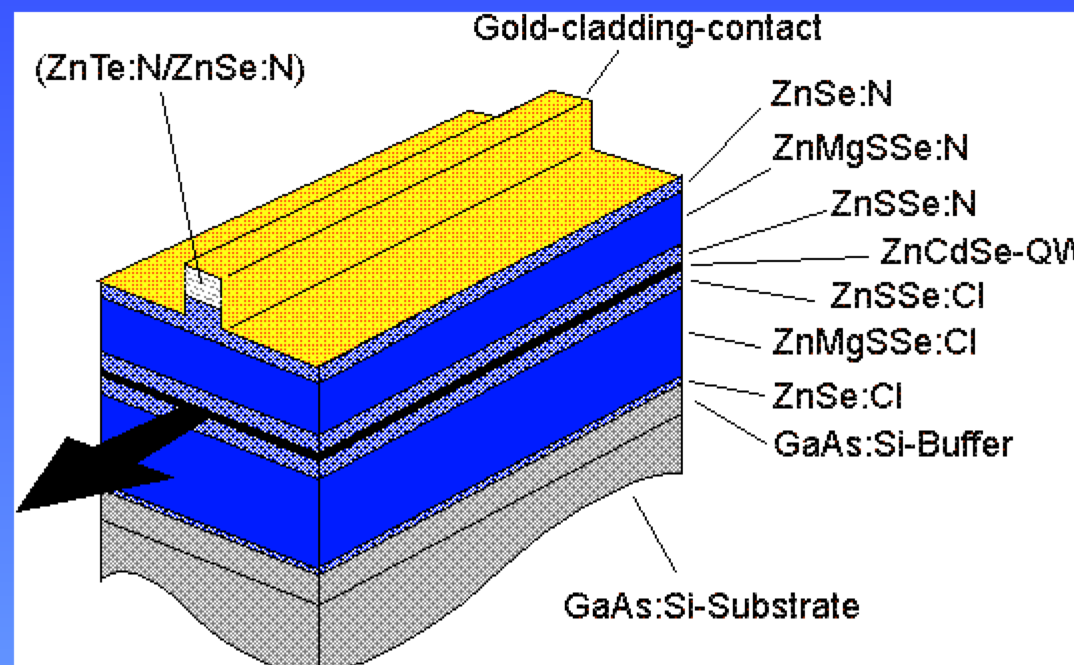
Production (Y) is ordering of components in the factory

Production (Y) ist entropy production in nature

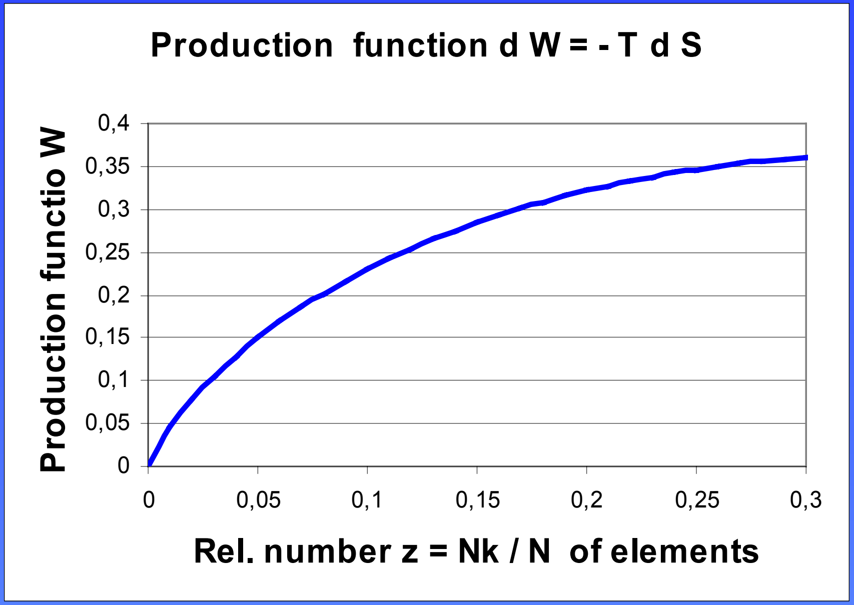
Kubic pn-GaN LED



Laser diode



Production function



Production factors x, y, z

Entropy production function

$$Y / T = -x \ln(x) - y \ln(y) - z \ln(z)$$

or

$$Y / T = -\ln(x^{x} y^{y} z^{z})$$

with $x + y + z = 1$

This replaces the Cobb Douglas production function

$$Y / T = -A (x^{\alpha} y^{\beta} z^{\gamma})$$

with $\alpha + \beta + \gamma = 1$ $\alpha, \beta, \gamma = ?$

The Carnot Process of Economic Growth and Wealth Distribution

Production costs: Material (K) and labor (T Δ S)

$$C_T = K + T \cdot \Delta S$$

Labor costs (w)

$$w = T \cdot \Delta S$$

The actual work (Δ S for building a car) is the same in different countries, but the wages are not. They depend on the mean standard of living T.

Application
The Carnot Process

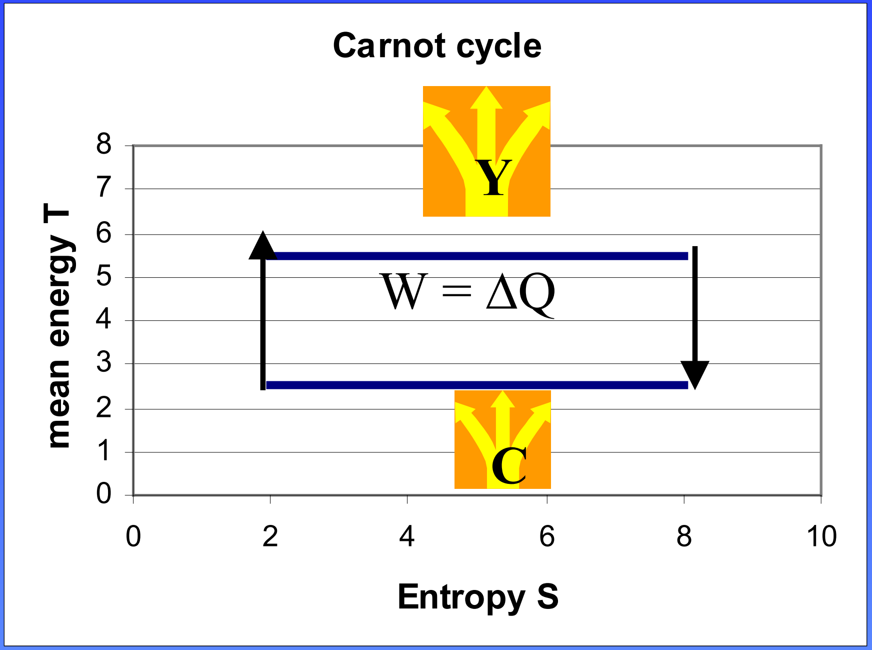
The Carnot Process of Economic Growth and Wealth Distribution

Carnot cycle of a motor, the fuel is oil

Second Law of Thermodynamics:

T_2

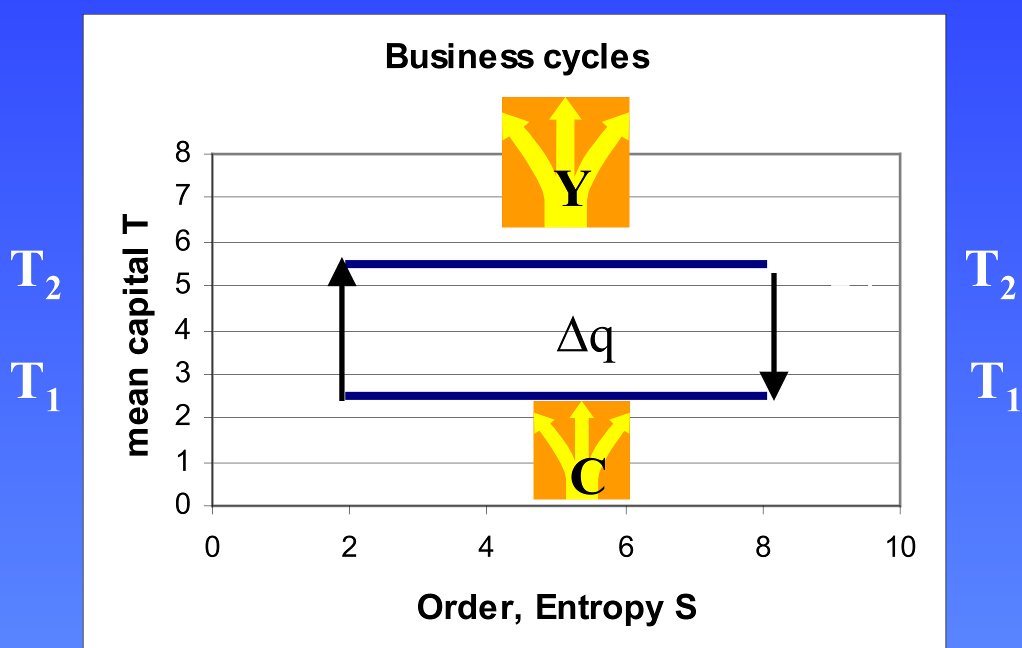
T_1



The Carnot Process of Economic Growth and Wealth Distribution

Carnot cycle of industry, the fuel is oil

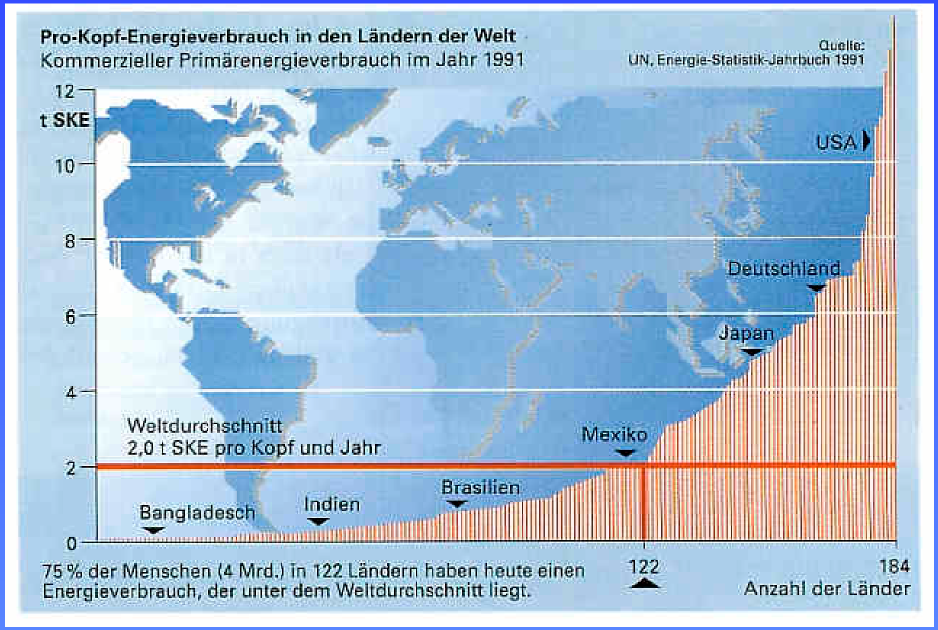
"Second Law" of Economics:



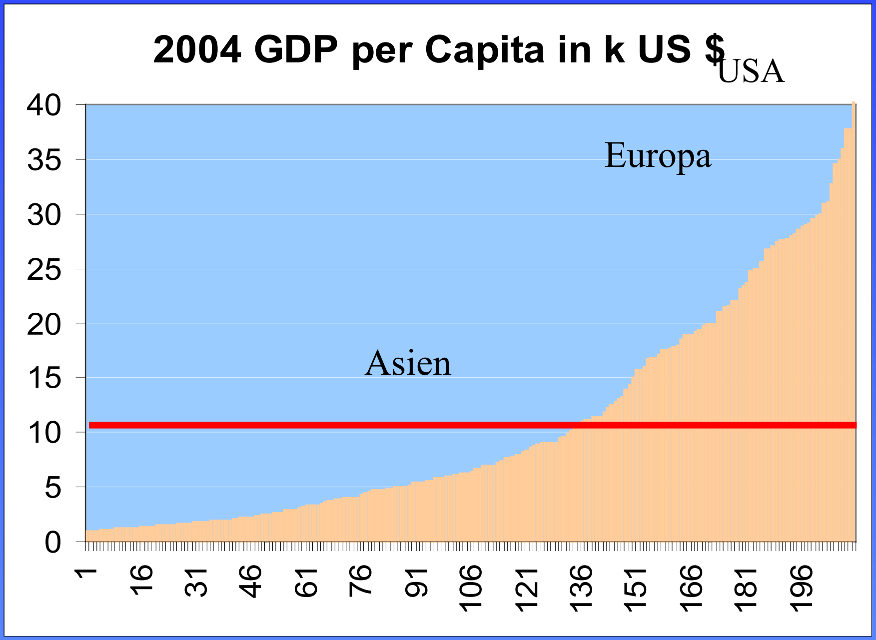
The Carnot Process of Economic Growth and Wealth Distribution

Carnot cycle of industry, energy consumption and GDP

"Second Law" of Economics:



World energy consumption / capita



World GDP / capita

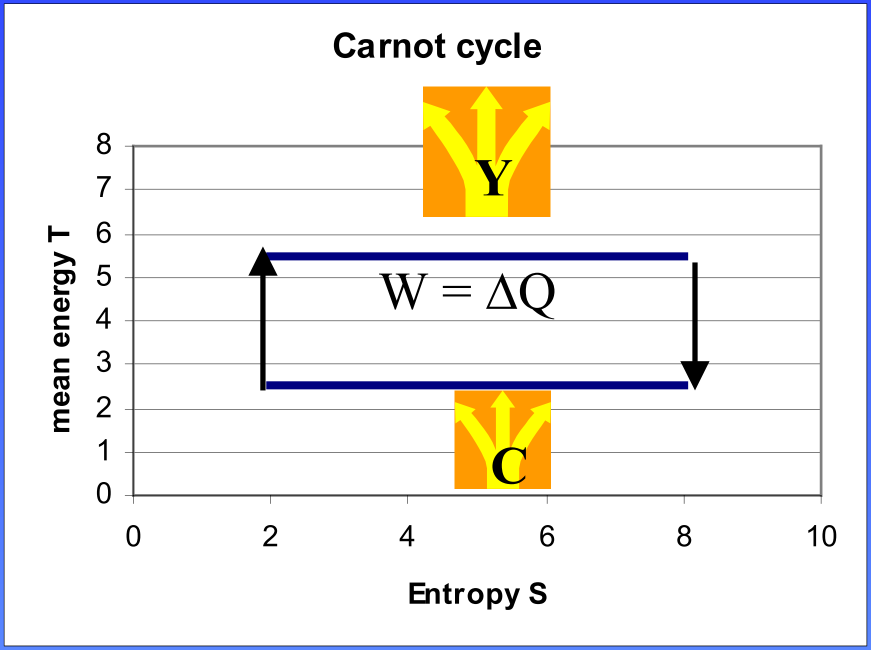
The Carnot Process of Economic Growth and Wealth Distribution

Carnot cycle of industry and economic growth

"Second Law" of Economics:

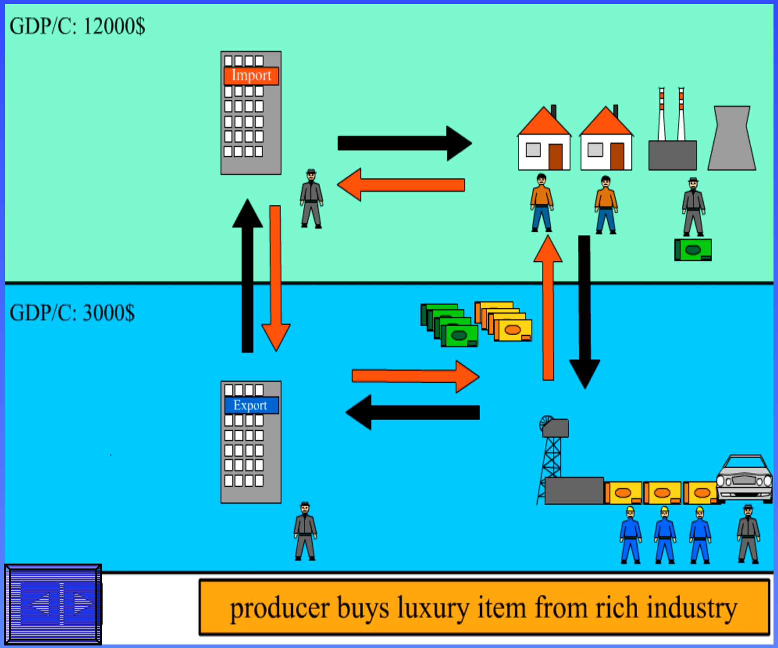
T_2

T_1



T_2

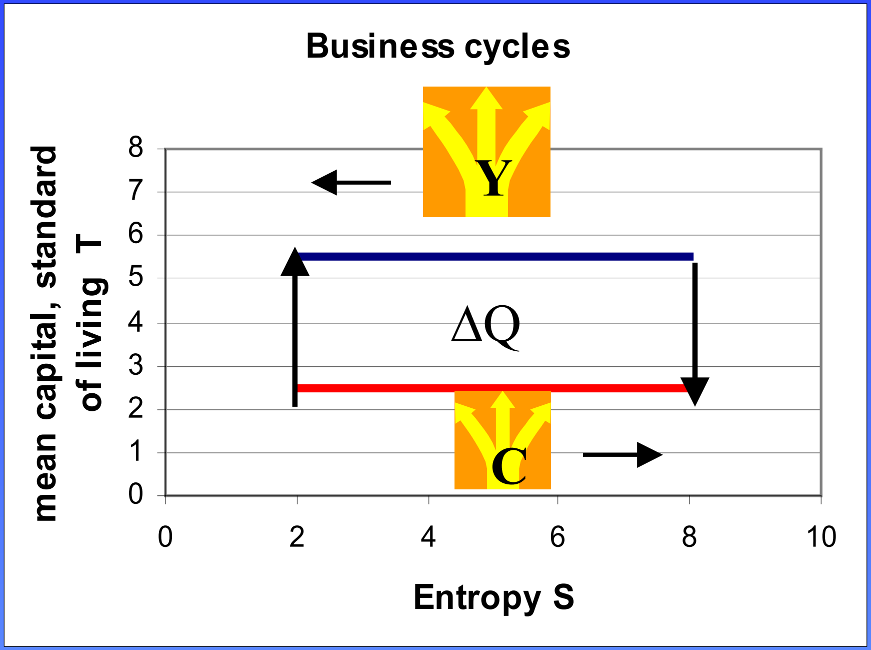
T_1



Carnot cycle of industry, farms, markets, banks and foreign trade

The Carnot Process of Economic Growth and Wealth Distribution

Carnot cycle of trade between India and Germany



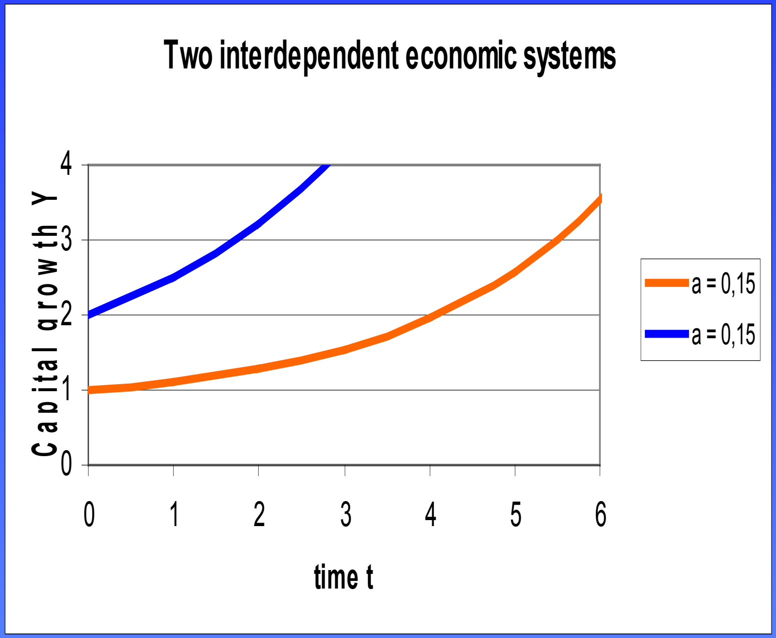
Income distribution

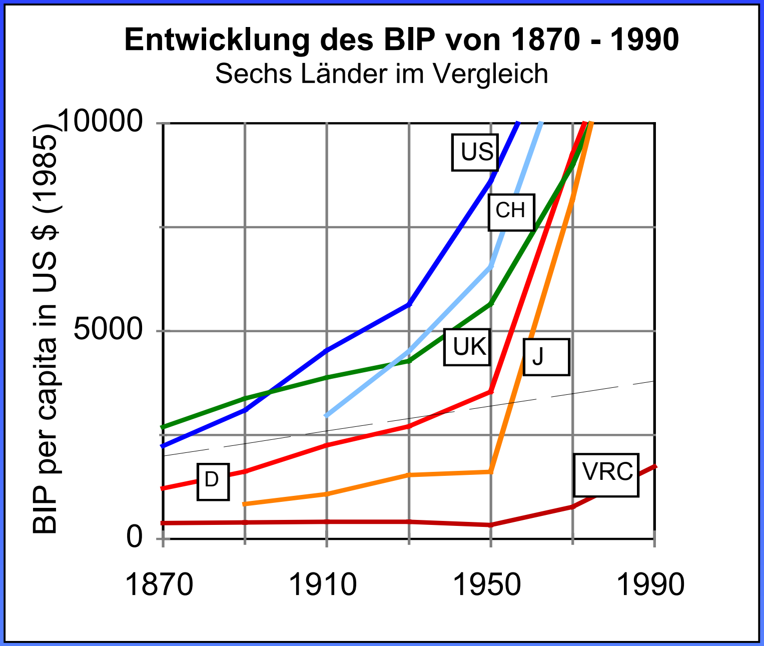
T(Germany)	**27.600 US$**
T(India)	**2.900 US$**

$$\eta = (T_2 - T_1)/T_1 = 9{,}5 : 1$$

The Carnot Process of Economic Growth and Wealth Distribution

Carnot cycle of industry and economic growth



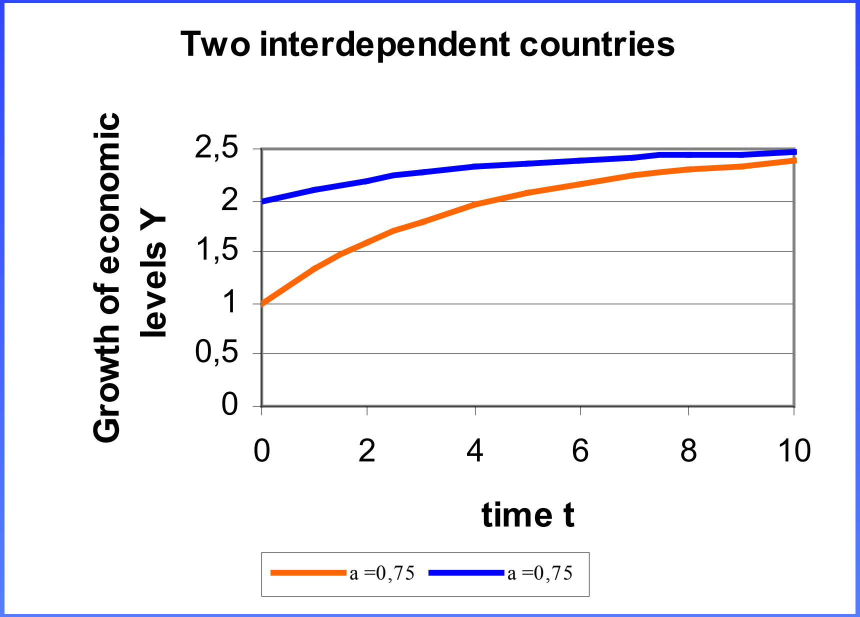


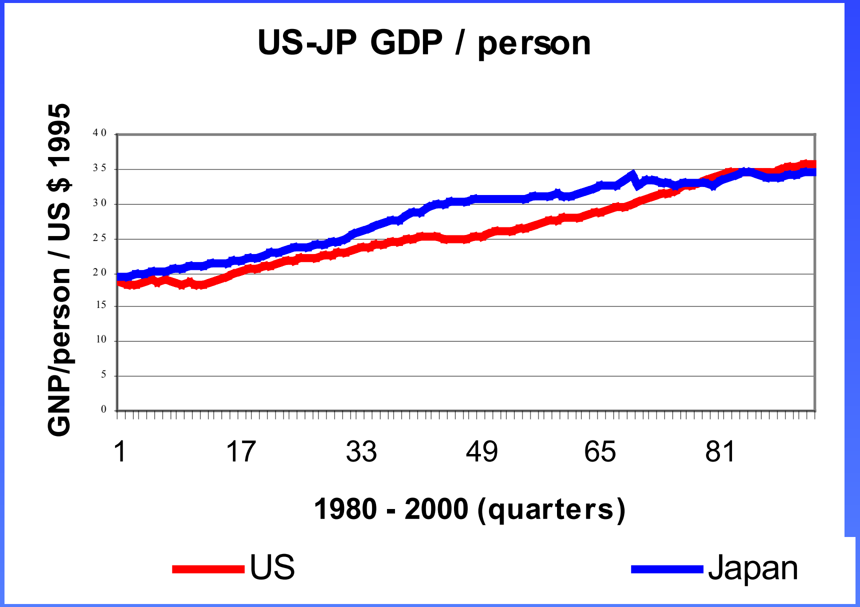
a = 0,15

US - Japan, US - D 1870-1970

The Carnot Process of Economic Growth and Wealth Distribution

Carnot cycle of industry and economic growth





a = 0,7

US - Japan, 1980 -2000

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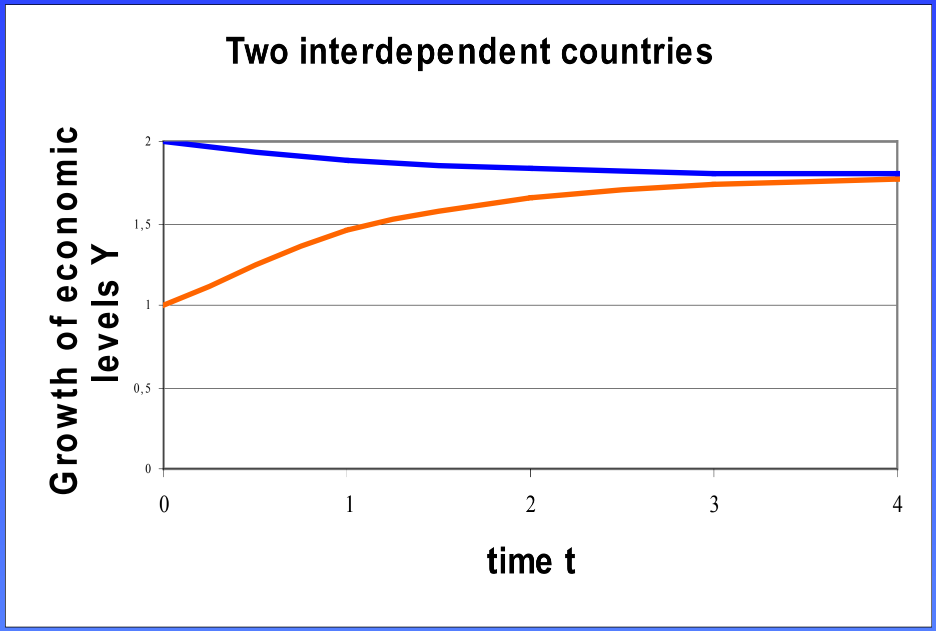
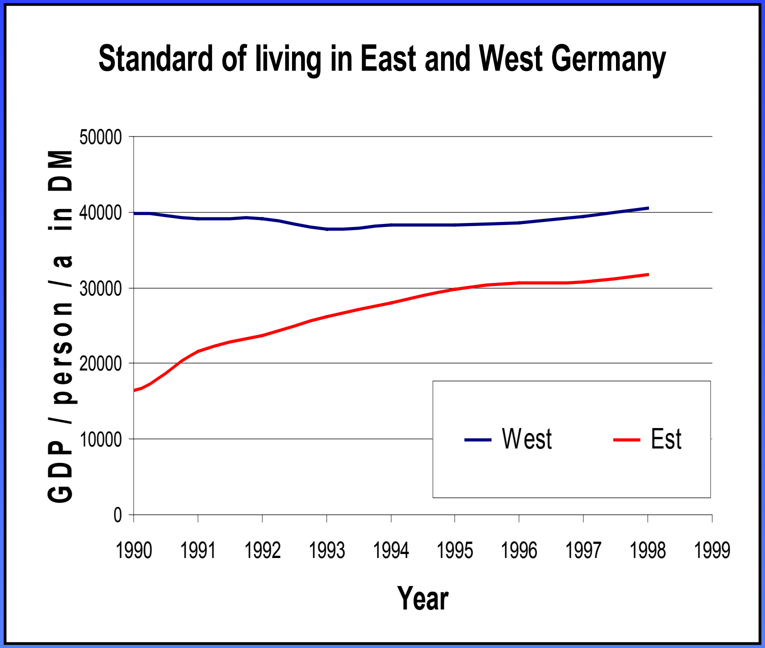
Carnot cycle of industry and economic growth



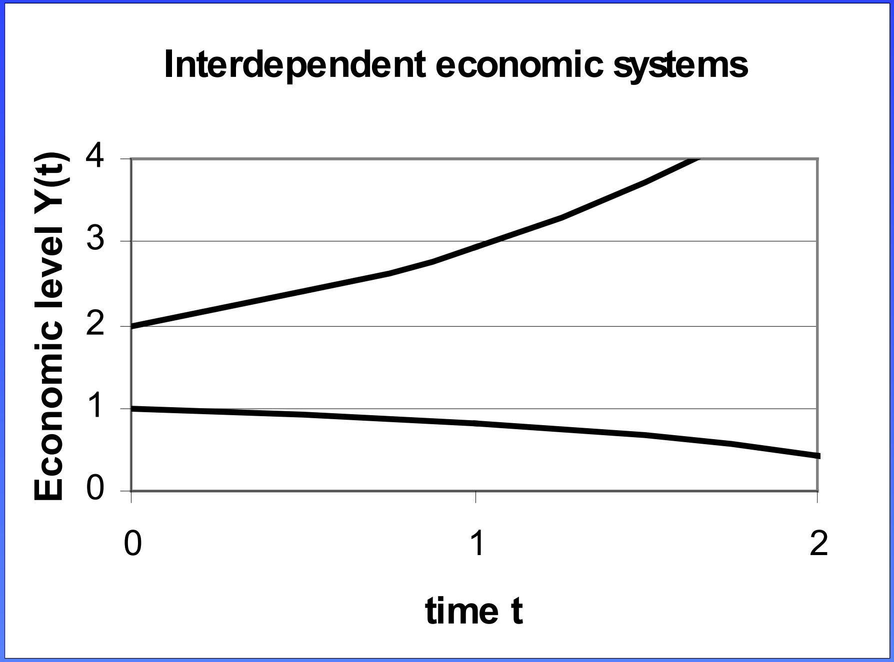


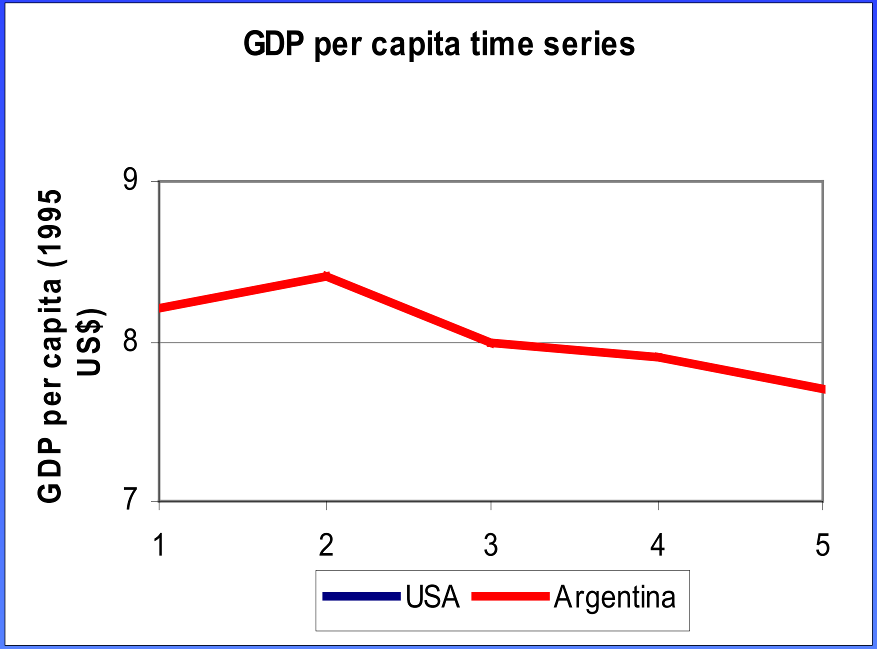
a = 1,35

D West - Ost, 1990 -2000

The Carnot Process of Economic Growth and Wealth Distribution

Carnot cycle of industry and economic growth





a = - 0,25

US - Argentinien, 1997 -2000

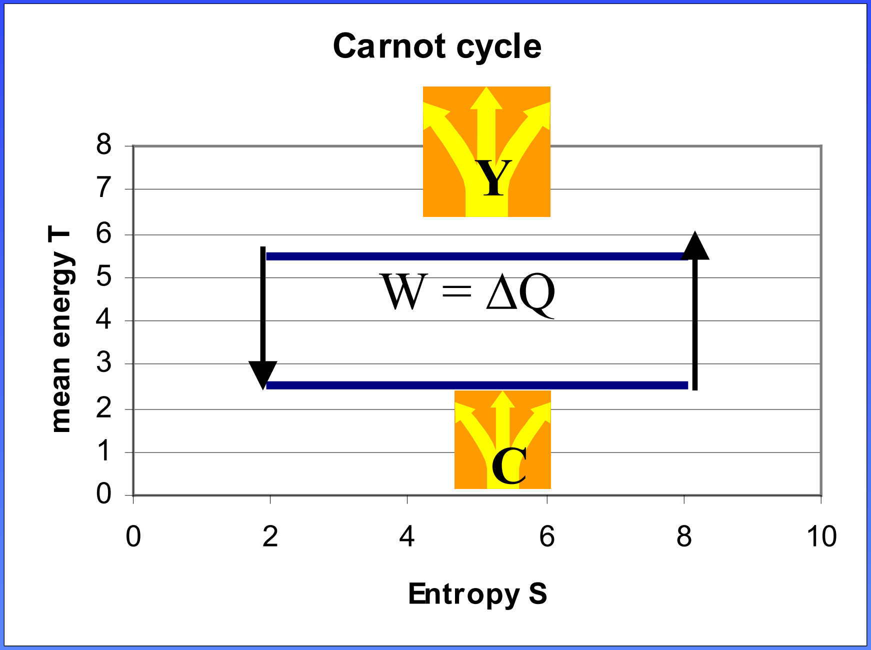
The Carnot Process of Economic Growth and Wealth Distribution

Carnot cycle of a heat pump

Second Law of Thermodynamics:

T_2

T_1



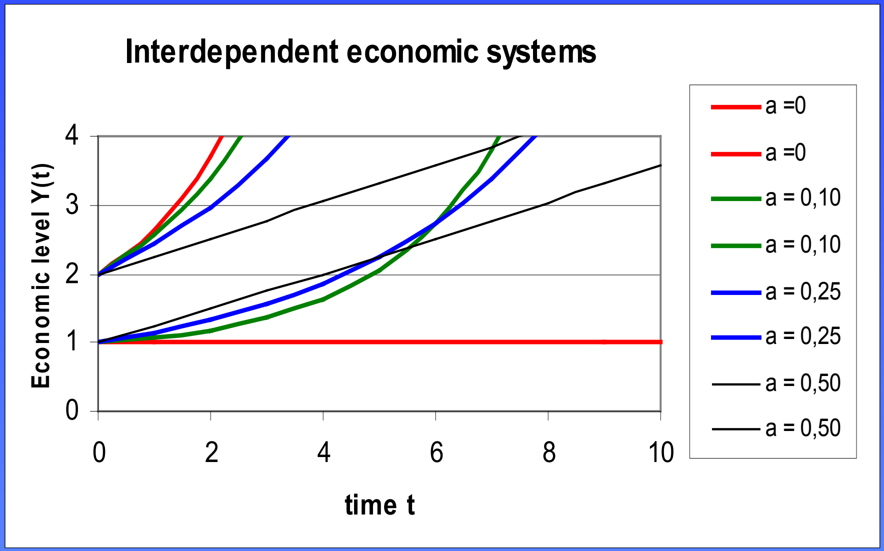
T_2 (warm house)

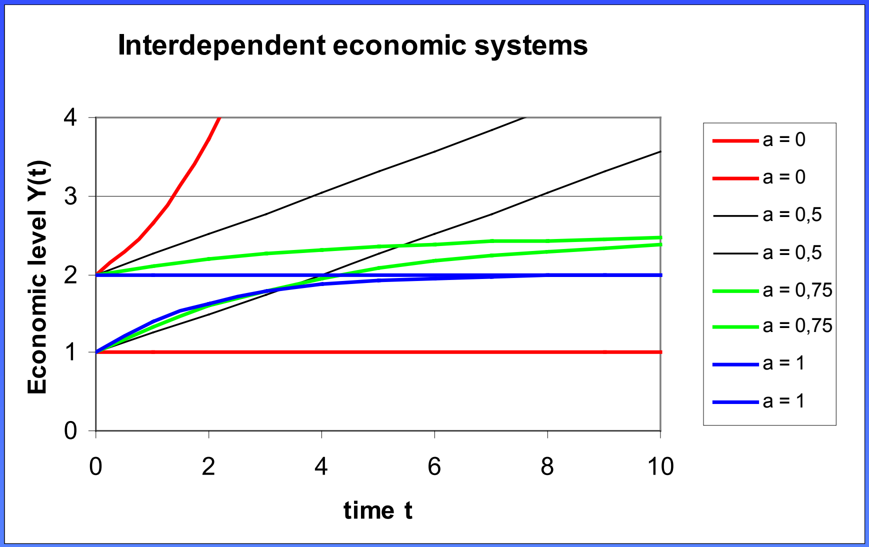
T_1 (cold river)

The Carnot Process of Economic Growth and Wealth Distribution

Carnot cycle of industry and economic growth

a: Profitanteil der Haushalte (Konsum C)



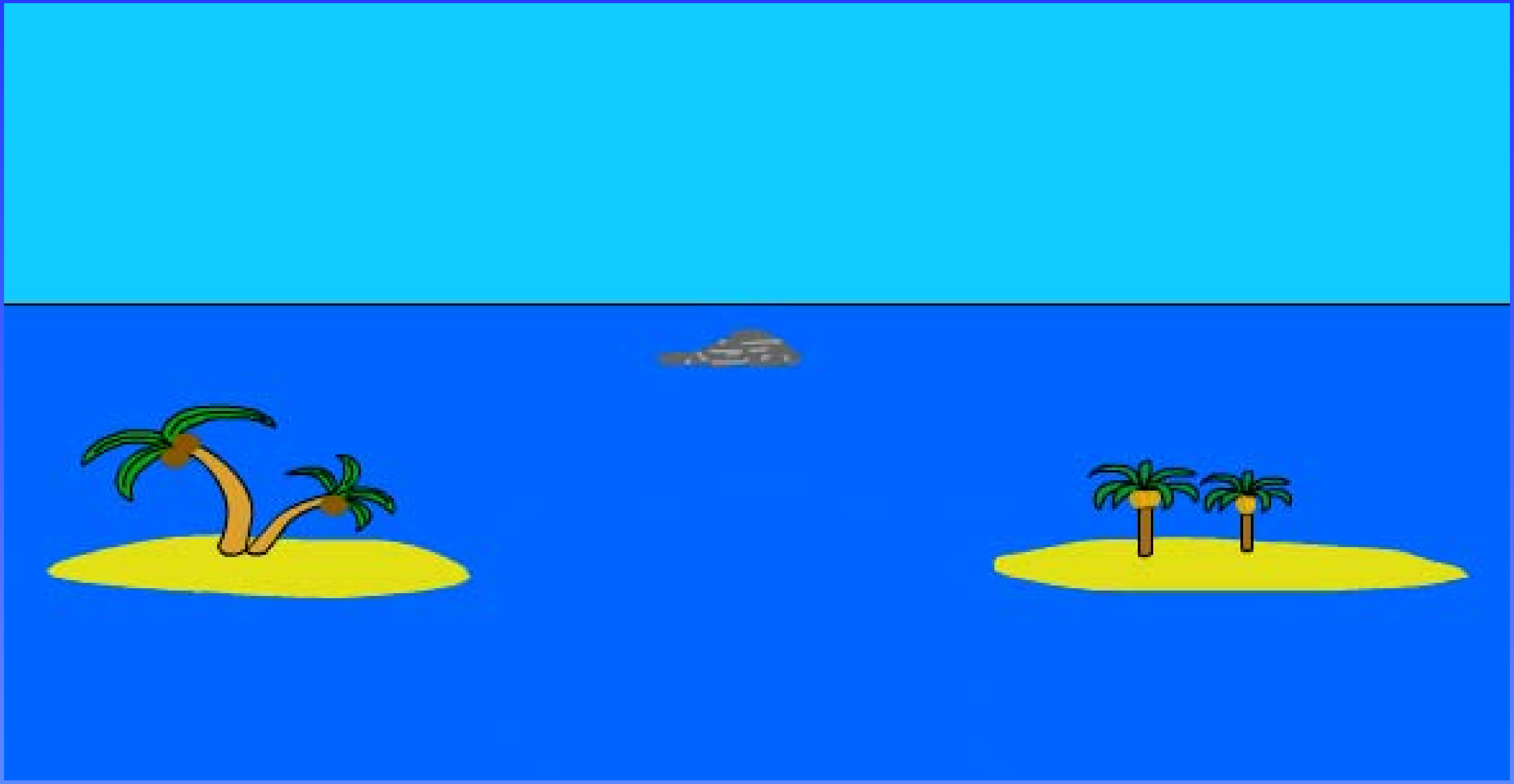


$0 \leq a \leq 0,5$

$0,5 \leq a \leq 1$

The Carnot Process of Economic Growth and Wealth Distribution

Carnot cycle of industry and economic growth

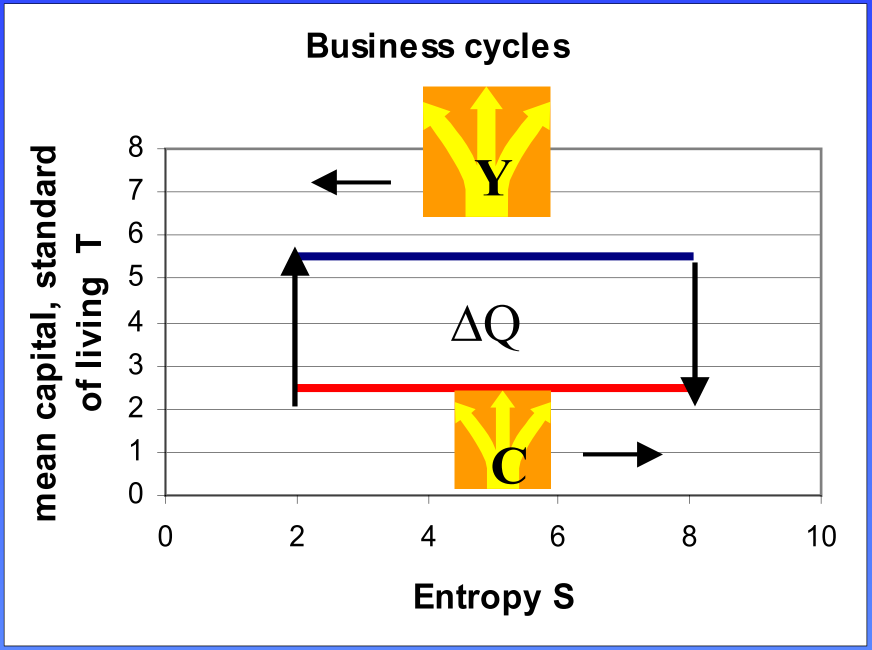


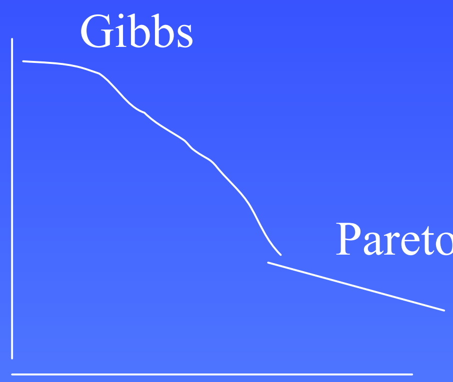
Economic Principles on Pacific Islands

Distribution of Wealth

The Carnot Process of Economic Growth and Wealth Distribution

Creation of ΔT by work in the Carnot cycle of industry

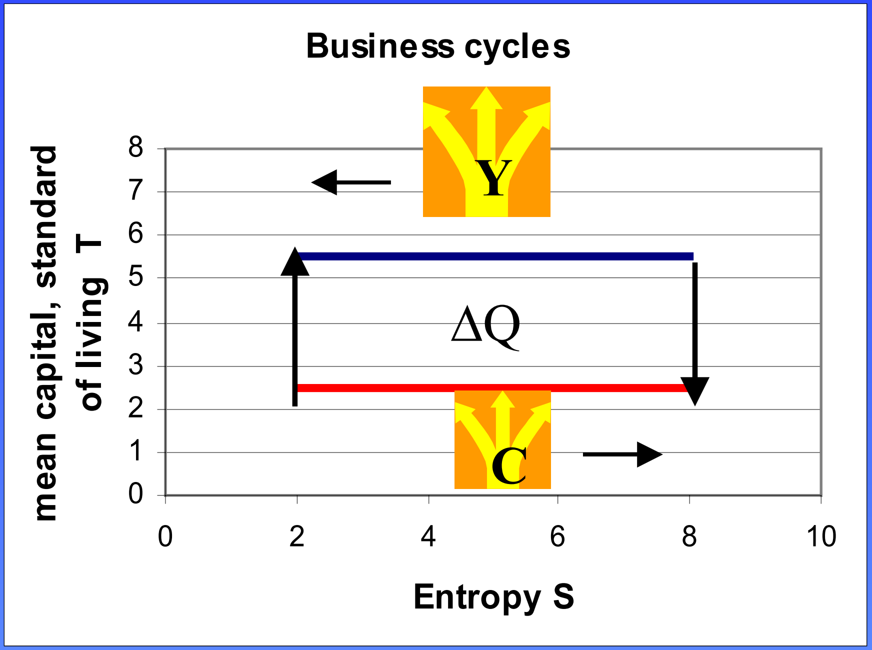


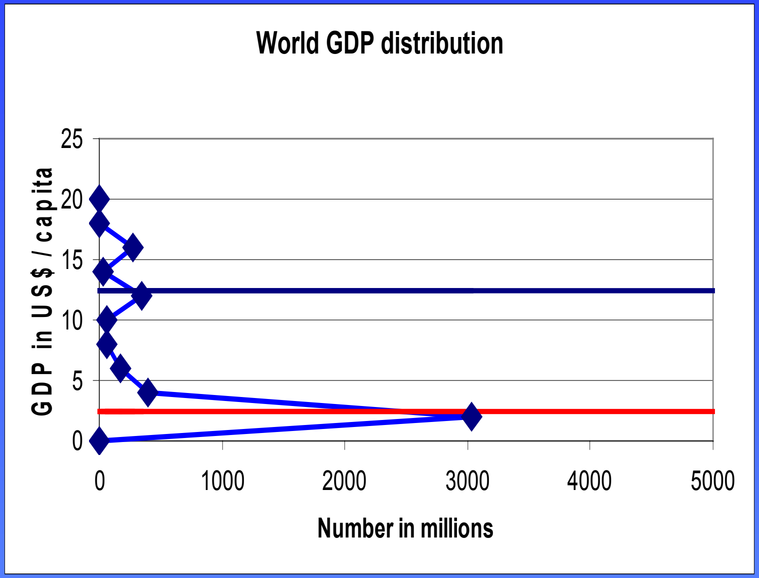


Income distribution

The Carnot Process of Economic Growth and Wealth Distribution

Creation of ΔT by work in the Carnot cycle of industry





World income distribution