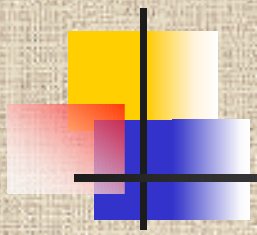

TERRESTRIAL ^{7}Be DECAY RATE AND ^{8}B SOLAR NEUTRINO FLUX

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Sun

Luminosity = 4×10^{26} Watts

Mass = 2×10^{30} Kg

Ancient Question

What powers the sun?

Chemical energy (coal burning)
Calculated life-time = 1500 years

Lord Kelvin: Gravitational contraction
Calculated Life-time = 50 - 100 million years
Life on earth much older

Modern Idea
Hans Bethe

Nuclear fusion at the solar core
Einstein's famous $E=mc^2$ formula

$4\ {}^{1}H \rightarrow {}^{4}He + \text{neutrinos} + 2e^{+} + \text{energy}$
Calculated life-time = 10 billion years

Density at solar core = 150 gm/cc
Temperature = 15 - 20 million °C.
Sophisticated solar model developed by John Bahcall et al.

Nuclear Reactions at the solar core

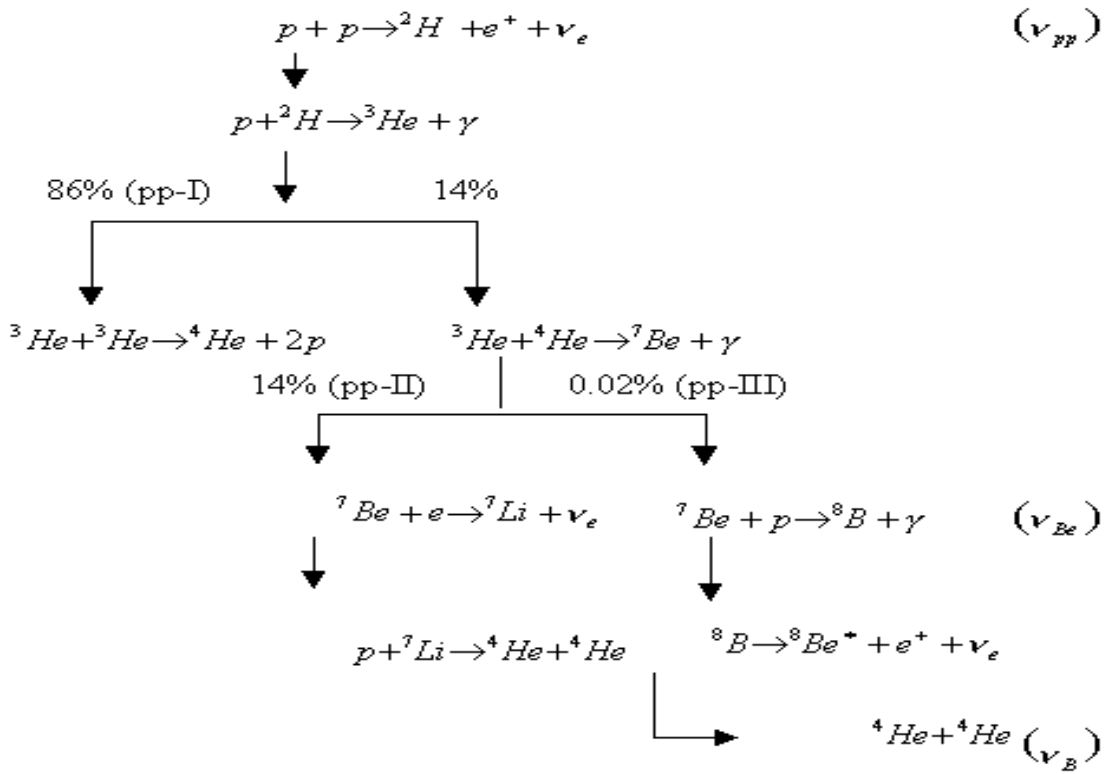


Fig (1.2) pp fusion chain reaction in the solar core and solar neutrinos produced

Davis Chlorine Experiment

- Homestake gold mine in South Dakota USA at a depth of 4200 meter water equivalent.

- 615 tons cleaning fluid (CCl_4) as target material

$$\nu_e + {}^{37}Cl \rightarrow e^- + {}^{37}Ar$$

Threshold energy 814 keV

- A few atoms of ^{37}Ar produced by neutrino interaction in the target were extracted every few months by purging the target with helium and added to the counting gas of a miniaturized, low background proportional counter.

(about 15 atoms out of total more than 10^{30} atoms in the tank)

- ^{37}Ar is 35 days by electron capture process (half-life, $\tau_{1/2}$= 35 days). The X-rays, Auger electrons produced subsequently were detected during counting period.
- **However only 1/3 of the expected neutrinos seen.**

H

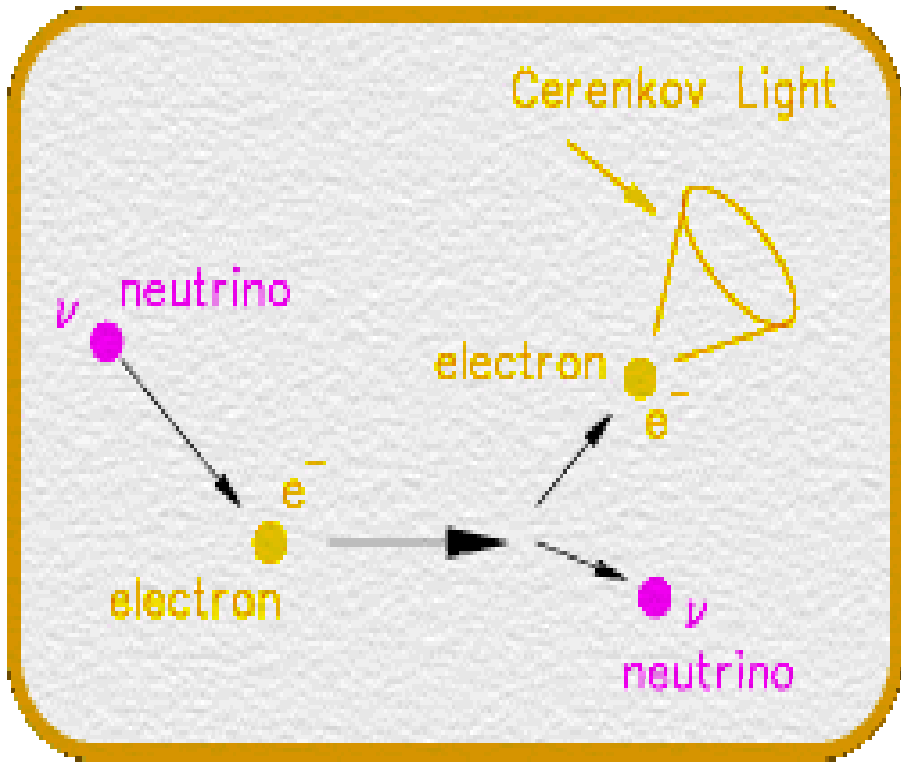
KAMIOKANDE and its upgraded SUPERKAMIOKANDE

•This water Čherenkov detector is located in the Kamioka mine in Japan.

•About 50 kilotons of ultrapure water has been used as the Čherenkov medium. Events seen by 11200 PMTs (each 50 cm diameter).

•Interaction is via elastic neutrino-electron scattering:

$$\nu + e^- \rightarrow \nu' + e^-$$



Sudbury Neutrino Observatory (SNO)

This imaging water Čerenkov detector is located at a depth of 6010 meter of water equivalent (6800 ft) in the Creighton Mine near Sudbury, Canada.

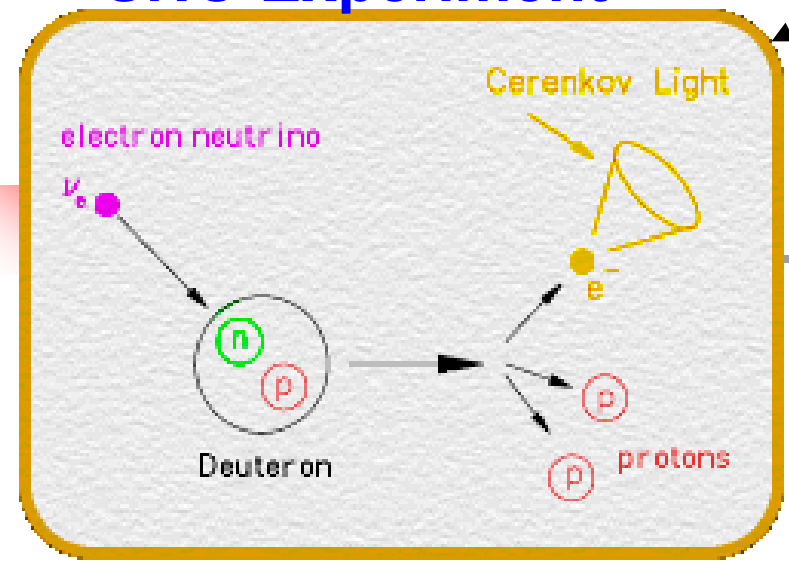
The target contains 1000 metric tons of ultra-pure D_2O in a 12 meter diameter spherical acrylic vessel. Vessel surrounded by very pure normal water. Events seen by 9600 PMT.

SNO Experiment

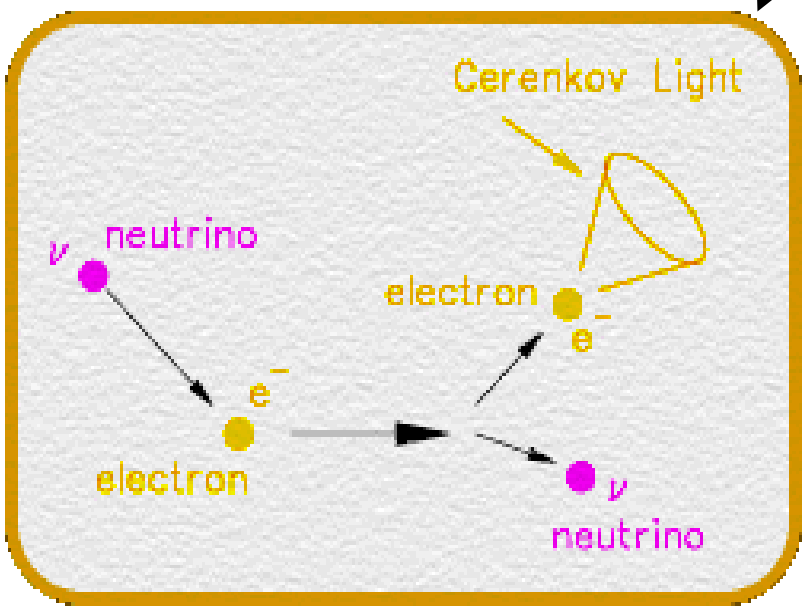
$$\nu_e + d \rightarrow p + p + e^- \quad \text{(CC)}$$

$$\nu_x + d \rightarrow p + n + \nu_x \quad \text{(NC)}$$

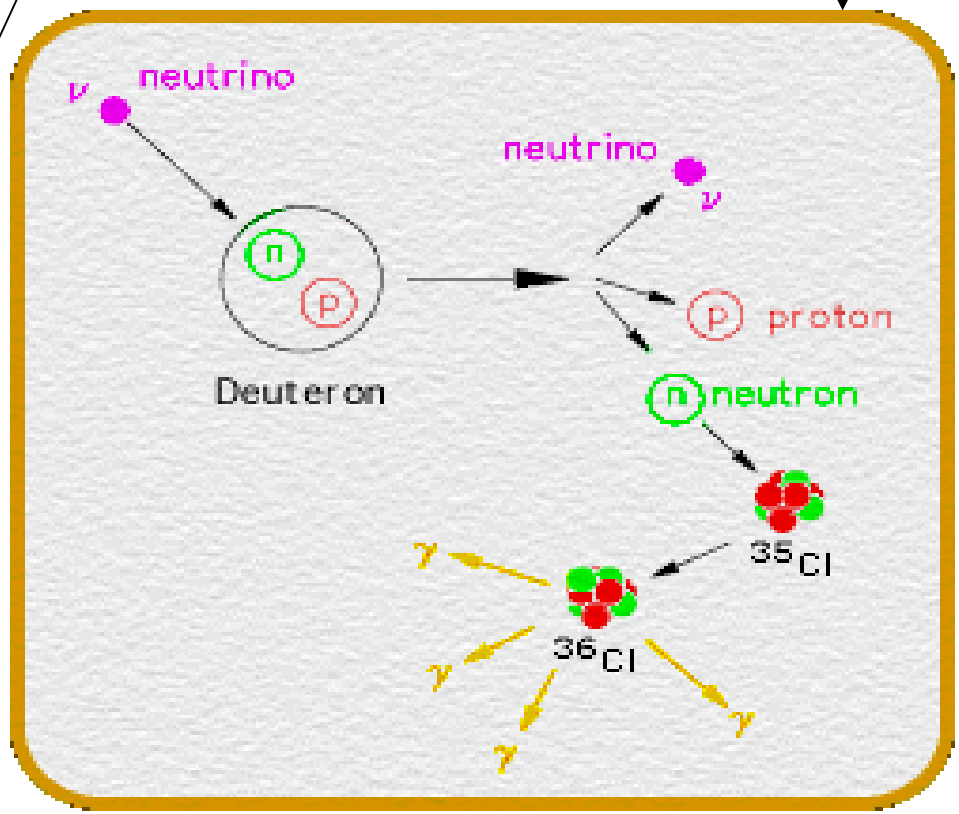
$$\nu_x + e^- \rightarrow \nu_x + e^- \quad \text{(ES)}$$



CC



ES



Superkamiokande and Sudbury Neutrino Observatory

Measured all flavors of ^{8}B solar neutrinos and found agreement with Standard Solar Model Calculations.

Recent ^{8}B solar neutrino flux result from SNO

$$\phi_{NC}^{SNO} = 5.21 \pm 0.27(stat.) \pm 0.38(syst.) \times 10^6\, cm^{-2} s^{-1}$$

SSM calculation

$$\phi_{SSM} = 5.05^{+1.01}_{-0.81} \times 10^6\, cm^{-2} s^{-1}$$

Agreement implies neutrino oscillation

Efforts going on to improve SSM calculation.

A few percent level effects also being considered.

Uncertainties in ^{8}B solar neutrino flux calculations

Sources of uncertainty	Uncertainty in ^{8}B neutrino flux
pp	0.040
$^{3}He^{3}He$	0.021
$^{3}He^{4}He$	0.075
$^{7}Be+p$	0.038
Z/X*	0.200
Luminosity	0.028
Opacity	0.052
Age	0.006
Diffusivity	0.040

* Z/X is the heavy element to hydrogen mass ratio

Effect of ^{7}Be decay rate on ^{8}B solar neutrino flux

Considering dynamic equilibrium condition at the center of the sun, the flux of ^{8}B solar neutrinos

$$\phi(^{8}B) \propto \frac{R(p)}{R(e)+R(p)}$$

Here R(p), R(e) are proton and electron capture rates of ^{7}Be at the solar core.

Since R(p)≈10^{-3}R(e), so $\phi(^{8}B) \propto \frac{1}{R(e)}$

R(e) at the solar core is calculated using nuclear matrix element extracted from terrestrial ^{7}Be decay rate measurement.

Calculation of R(e) at the solar core

$$R(e)_{sun} = R(e)_{lab} A^{-1} \left(\frac{2}{\pi k T} \right)^{1/2} \alpha Z n_e$$

T,n_e are solar temperature, electron density at the solar core.

α,Z fine structure constant, atomic number of ^{7}Be.

A is atomic overlap factor for terrestrial ^{7}Be.

$$A = \frac{1}{4\pi^2} \left[\{\psi_{1s}(0)\}^2 + \{\psi_{2s}(0)\}^2 \right]$$

$\psi_{1s}(0)$ and $\psi_{2s}(0)$ are electronic wave functions at the nucleus.

A computed assuming ^{7}Be has two full 1s and 2s electrons.

This assumption questionable for all terrestrial measurements done so far.

Change of ^{7}Be decay rate in different environments

$$^{7}Be + e^{-} \rightarrow {}^{7}Li + \nu_e$$

Decay rate $\propto$ electron density at ^{7}Be nucleus

Decay rate of ^{7}Be in different beryllium compounds (^{7}BeO, $^{7}BeF_2$) measured and up to 0.2% change in decay rate found.

Decay rate of implanted ^{7}Be in Al, LiF, Au, Ta, Graphite(C) measured and 0.4% - 0.5% change in decay rate found.

We measured change in decay rate of ^{7}Be in Au and Al_2O_3 and found 0.72% change in decay rate.

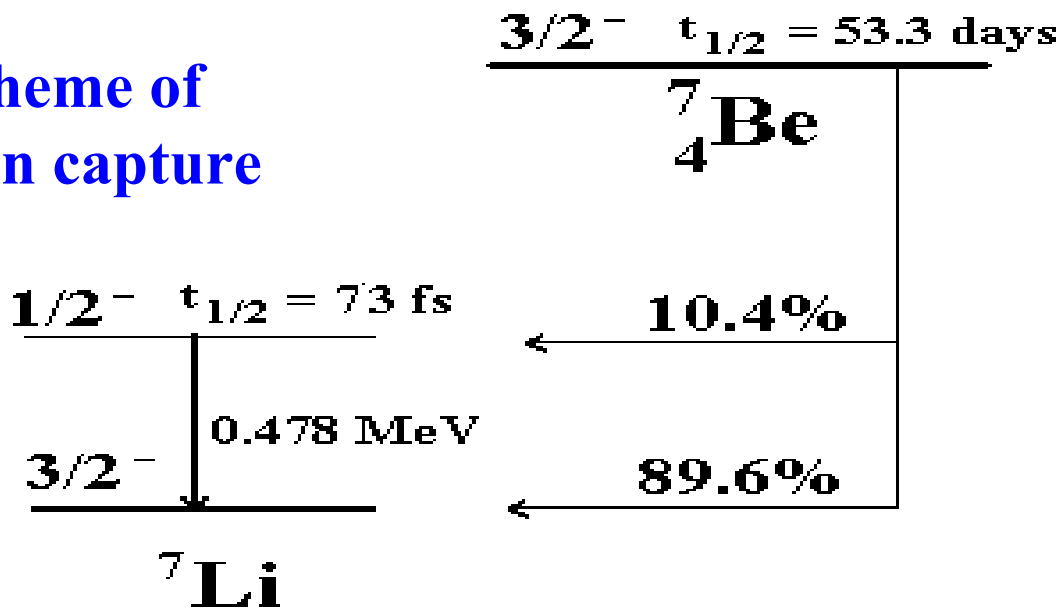
^{7}Be implanted in Au and Al_2O_3

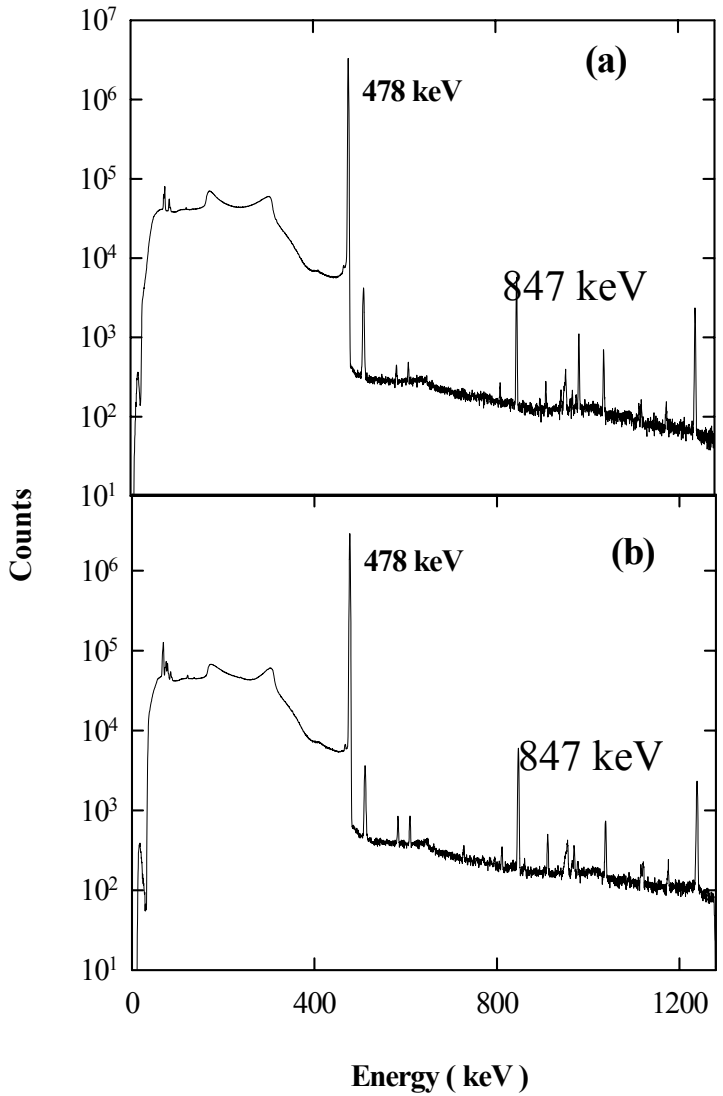
^{7}Be was produced by bombarding a 250 μg/cm^2 thick lithium fluoride (LiF) target with a 7 MeV proton beam obtained from Variable Energy Cyclotron Centre, Kolkata.

Reaction *via* which ^{7}Be was produced via

$$p + {}^{7}Li \rightarrow {}^{7}Be + n$$

Decay scheme of ^{7}Be electron capture





γ-ray spectra from decay of ^{7}Be implanted in
(a) an Al_2O_3 pellet
(b) an Au foil

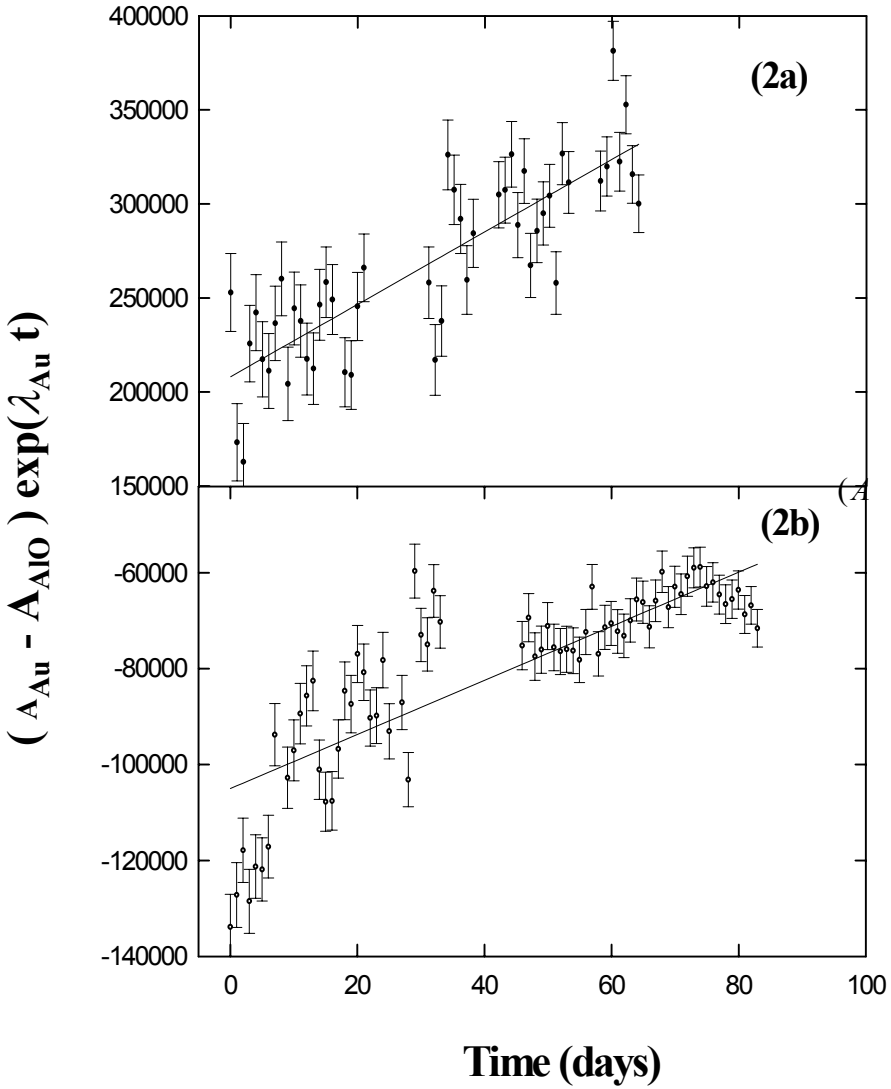
Differential measurement

$$(A_{Au} - A_{AlO})e^{\lambda_{Au}t} =$$

$$(A^0_{AlO}\Delta\lambda)t + (A^0_{Au} - A^0_{AlO})$$

$$\lambda_{AlO} = \lambda_{Au} + \Delta\lambda$$

Plot of $(A_{Au} - A_{AlO})e^{\lambda_{Au}t}$ versus time



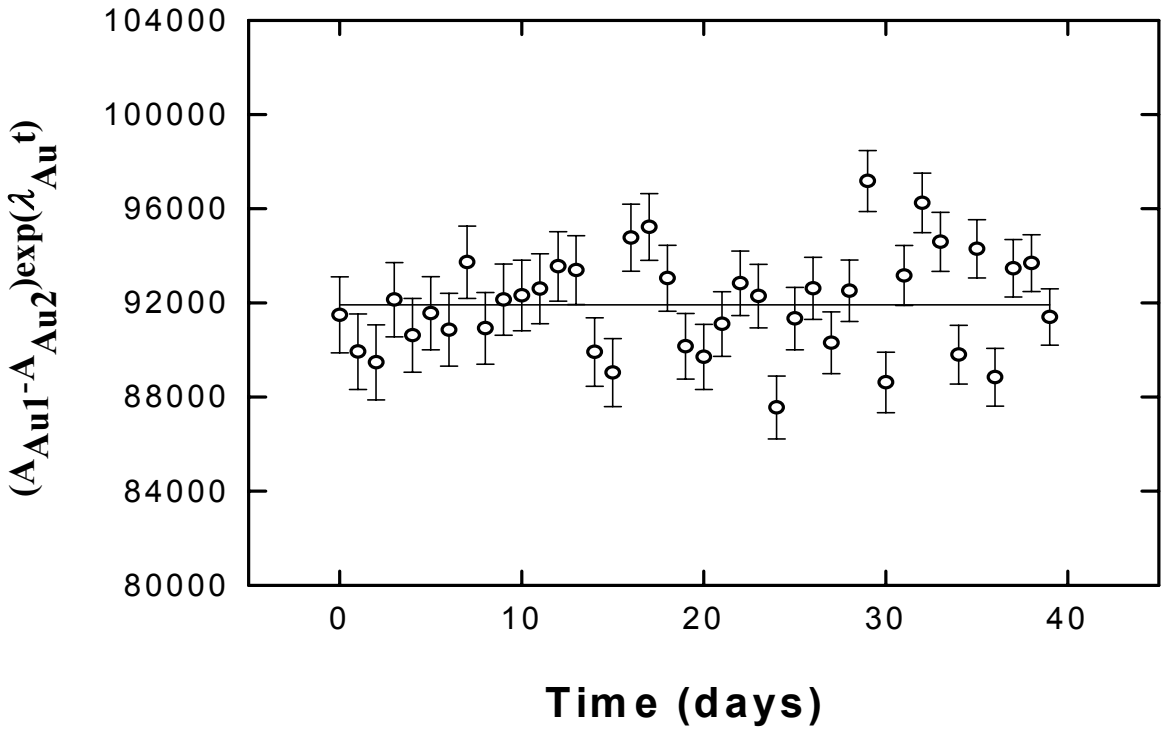
$$\frac{\Delta\lambda}{\lambda_{Au}} = (0.00705 \pm 0.00072)$$

$$\frac{\Delta\lambda}{\lambda_{Au}} = (0.0078 \pm 0.0016)$$

Result

$$\frac{\Delta\lambda}{\lambda_{Au}} = (0.0072 \pm 0.0007)$$

Check of systematic error



Plot of $(A_{Au1}-A_{Au2})exp(\lambda-_{Au}t)$ **versus time**

Experiment -2

^{7}Be implanted in Au and Zn

- ^{7}Be was produced by an inverse process i.e. bombarding a proton rich target with heavy ion ^{7}Li beam obtained from BARC+TIFR peletron machine at Mumbai.

^{7}Be was produced *via* nuclear reaction

$$^{7}Li + p \rightarrow {}^{7}Be + n$$

Experiment-3

^{7}Be implantation in Au, C_{60} and Cd using a pure ^{7}Be beam

The recoiled ^{7}Be ions produced in the reaction were separated from primary ^{7}Li beam by using a recoil mass spectrometer called Heavy Ion Reaction Analyser (HIRA) system NSC Delhi.

Experimental result

Difference between the half-lives of ^{7}Be implanted in	Percentage increase in Half-life of ^{7}Be in 1st medium compared to that in 2nd medium in column-1
Au and Al_2O_3	(0.72±0.07)%
Au and Zn/Cd	(0.57±0.32)%
Au and Fullerene (C_{60})	(0.08±0.22)%

Half-life of ^{7}Be in Au =(53.328± 0.082) days (Expt 2)

Half-life of ^{7}Be in Au = (53.311± 0.041)days (Norman et al.)

Half-life of ^{7}Be in Au = (53.60±0.19) days (Expt 3)

Other Measurements

Older measurements:

Johlige et al. measured ^{7}Be decay rates in different compounds. 0.2% difference (Phys. Rev **C2**, 1616 (1970)).

Recent Measurements:

Norman et al. measured half-lives of ^{7}Be in Au, graphite, tantalum. 0.38% change in decay rate found. (Phys. Lett. **B519**, 15 (2001))

Ohtsuki et al. measured decay rate of ^{7}Be @C_{60}. 1.2% difference in decay rate. (PRL, **93**, 112501(2004)).

Liu et al. measured decay rate of ^{7}Be in Au and Be. 0.12% difference seen. (Chin. Phys. Lett. **20**, 829 (2003)).

Qualitative understanding of decay rate results

Electron affinity of the host medium is primarily responsible for changing the number of valence 2s electrons of ^{7}Be. As a result, the decay rate of implanted ^{7}Be changes in different host media.

Decay rate of ^{7}Be found to be slowest in Au.
Electron Affinity of Au = 2.3 eV

Decay rate faster in Al, graphite, Al_2O_3.

Need to consider lattice structure also.

Difference between the half-lives of (a) ^{7}Be implanted in	Electron affinity* in (eV) 1st medium	2nd medium	Observed half- life difference $\frac{\Delta\lambda}{\lambda}$ ×100%	References on e half-life measured
Au and Al	2.308	0.441	(0.27±0.15)%	Norman+2001, Lagoutine+75
Au and Ta	2.308	0.322	(0.22±0.13)%	Norman+2001
Au and C(graphite)	2.308	1.25	(0.38±0.09)%	Norman+2001
Au and Cd/Zn	2.308	0-negative	(0.57±0.32)%	This work
Ta and C(graphite)	0.322	1.25	(0.17±0.11)%	Norman+2001
Au and LiF	2.308	~0	(0.36±0.15)%	Norman+2001, Jaeger+96
Al and LiF	0.441	~0	(0.10±0.20)%	Lagoutine+75, Jaeger+96
Au and Al_2O_3	2.308	~0	(0.72±0.07)%	This work
Au and C_{60}	2.308	2.6	(0.08±0.22)%	This work
Au and ^{9}Be	2.308	~0	(0.02±0.06)%	Liu+2003
Ta and Al	0.322	0.441	(0.05±0.13)%	Norman+2001, Lagoutine+75
C_{60} and C(graphite)	2.6	1.25	(0.31±0.13)%	This work, Norman+2001
(b) ^{7}Be compounds $^{7}BeF_2$ and ^{7}BeO	Fluorine- 3.40	Oxygen- 1.46	(0.1375±0.0053)% (0.0609±0.0055)% (0.1130±0.0058)%	Leininger+49 Kraushaar+53 Johlige+70

TB-LMTO calculations for ^{7}Be in a medium or forming compounds

Tight binding linear muffin-tin orbital method calculation is a first principle density functional calculation.

Initial Ansatz: Charge distribution spherical around each atom.
Kohn-Sham equation solved self-consistently
while minimizing total energy.
LMTO basis states used.

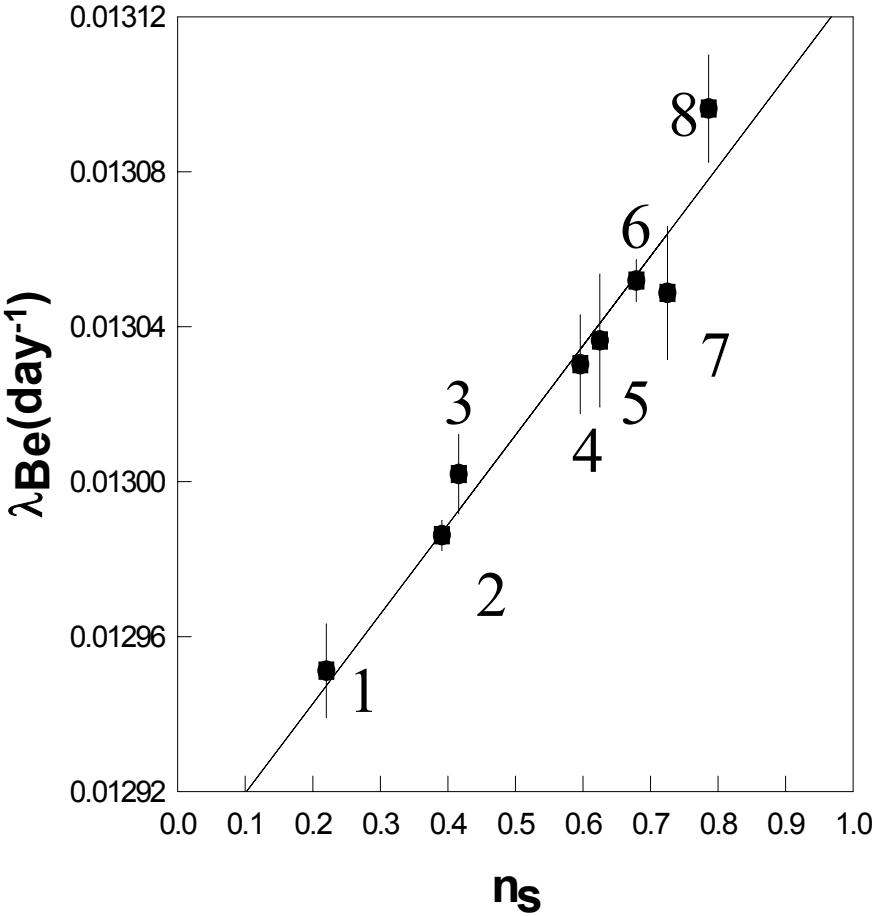
$$V_{MT}(r) = V_i(r_i) + \sum_R V_R(r_R) \equiv V_o + \sum_R v_R(r_R)$$

Input: Lattice dimensions, symmetry group, atomic structures.

Output: $\left|\langle \psi_{total} | \psi_{Be2s} \rangle\right|^2$ computed for ^{7}Be.

This represents average number (n_s) of 2s electrons of ^{7}Be in a host medium or compound.

Plot measured ^{7}Be decay rate (λ_{Be}) versus n_s calculated from LMTO code.



Decay rate difference between ^{7}Be(n_s=2) and ^{7}Be (n_s=0) is =3.4%. Agrees with Hartree's calculation(3.31%).

Measured terrestrial ^{7}Be decay rate lower than neutral ^{7}Be decay rate by 2% - 2.7%.

Difference in Half-life of ^{7}Be in	Percentage increase of half-life of ^{7}Be in 1st medium compared to that in 2nd medium		Method of ^{7}Be implantation in the hosts
	Experimental Value	Calculated value	
Au and Al_2O_3	(0.72±0.07)%	0.61%	Both using proton irradiation
Al and LiF	(0.1±0.2)%	0.17%	Both using proton irradiation
Au and ^{9}Be	(0.02±0.06)%	0.04%	
Au and Ta	(0.22±0.13)%	0.30%	Both using heavy ion ^{7}Li irradiation
Au and C(graphite)	(0.38±0.09)%	0.44%	Both using heavy ion ^{7}Li irradiation
Ta and C(graphite)	(0.17±0.11)%	0.14%	
Au and Al	(0.27±0.15)%	0.35%	Using ^{7}Li irradiation in 1st medium and proton irradiation in 2nd medium
Au and LiF	(0.36±0.15)%	0.51%	
Au and Cd/Zn	(0.57±0.32)%	0.42%	Both using ^{7}Be beam
^{7}BeO and $^{7}BeF_2$ (hex)	(0.1375±0.0053)% (0.0609±0.0053)%	0.124%	The compounds were prepared following chemical processes
^{7}BeO and $^{7}BeF_2$ (tetra)	(0.1130±0.0058)%	0.115%	

Effect of medium on L/K electron capture ratio of ^{7}Be

If indeed n_s is significantly different from 2, then unlike small effect on decay rate, large decrease of L/K electron capture ratio expected.

Recently, L/K capture ratio of ^{7}Be in HgTe measured
P. Voytas et al., Phys. Rev. Lett 88, 012501 (2002).
Expt L/K ratio = 0.040±0.006
Theoretical L/K ratio = 0.09

For ^{7}Be in HgTe, n_s=1.155
Zeroth order correction factor =1.155/2.0 = 0.577
Then theoretical L/K ratio = 0.052

Bahcall considering a dense hot plasma at the solar core derived the electron capture rate of ^{7}Be as a function of electron temperature and concentration and obtained

$$R(e)_{star} = (2/\pi kT)^{1/2} G_V^2 \alpha Z n_e q^2 \xi$$

The terrestrial (laboratory) decay rate of ^{7}Be is given by

$$R(e)_{lab} = G_V^2 A(q_o^2 \xi_o + q_1^2 \xi_1) = G_V^2 A q^2 \xi$$

$$R(e)_{sun} = R(e)_{lab} A^{-1} \left(\frac{2}{\pi kT}\right)^{1/2} \alpha Z n_e$$

$$A = \frac{1}{4\pi^2}\left[\{\psi_{1s}(0)\}^2 + \{\psi_{2s}(0)\}^2\right]$$

Observed decay rate change of ^{7}Be in different media
&
Measured L/K electron capture ratio

^{7}Be in a medium loses significant fraction of its 2s electrons.

Overlap factor *A* decreases by 2% to 2.7% in different media.

$$R(e)_{sun} = R(e)_{lab} A^{-1} \left(\frac{2}{\pi k T} \right)^{1/2} \alpha Z n_e$$

R(e) increases by 2 – 2.7% .

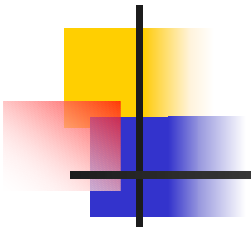
Since $\phi(^{8}B) \approx \frac{1}{R(e)}$, calculated $\phi(^{8}B)$ decreases by 2 – 2.7%.

Summary

- **Experimentally measured decay rate change of ^{7}Be in different media (Au, Al_2O_3, Cd, Zn, fullerene (C_{60})).**

- **Our data and other available results on the change of ^{7}Be decay rate were qualitatively and quantitatively understood.**

- **Calculated ^{8}B solar neutrino flux should decrease by 2%-2.7%**



Thankyou

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P. Sugathan
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S. Nath

Exchange Correction Factor for L/K Capture Ratio

Direct L capture

One 2s electron absorbed by ^{7}Be

(both) $2s \rightarrow 2s';1s \rightarrow 1s'$

Exchange L capture

One 1s electron absorbed by ^{7}Be

one $2s \rightarrow 1s';1s \rightarrow 1s';2s \rightarrow 2s'$

$$\frac{\lambda_L}{\lambda_K} = \left(\frac{\lambda_L}{\lambda_K}\right)^0 \left\{\frac{1-\left(R_{1s}(0)/R_{2s}(0)\right)\langle 1s'|2s\rangle}{1-\left(R_{2s}(0)/R_{1s}(0)\right)\langle 2s'|1s\rangle}\right\}^2$$

Expected L/K electron capture ratio of ^{7}Be in other media

^{7}Be implanted in	Correction factor for in-medium effect	In-medium effect corrected (L/K) ratio	
		Bahcall's calculations	Vatai's calculations
HgTe	0.577	0.0519	0.0635
Al_2O_3	0.404	0.0363	0.0475
^{9}Be	0.4155	0.0374	0.0457
LiF	0.3659	0.0333	0.0406
Al	0.344	0.0310	0.0378
Cd	0.3385	0.0305	0.0372
Zn	0.3295	0.03	0.0362
Ta	0.2986	0.027	0.0328
Au	0.208	0.0187	0.0229

TB-LMTO method: A schematic view

First principle density functional calculation

Hohenberg-Kohn Theorem:

Total enery is a functional of e^- density.
Energy functional $E[\rho(r)]$ is minimum for ground state density ρ_0.

Total Energy functional for a system of electrons in presence of interacting ions

$$E[\rho(r),\{R\}] = E_{el}[\rho(r),\{R\}] + E_{ion}[\{R,R'\}]$$

$$= T_s[\rho] + E_{xc}[\rho] + E_{ES}[\rho,\{R\}]$$

$$= T_s[\rho] + E_{xc}[\rho] + E_{ES}[\rho, \{R\}]$$

$T_s[\rho]$ the kinetic energy (K.E.) of non-interacting electron gas of same density as that of the actual system

$E_{xc}[\rho]$ the exchange-correlation energy which depends on the relative orientation of the spin of the electrons

$E_{ES}[\rho]$ the total electrostatic energy, i.e. energies due to classical electron-electron Coulomb, external potential and ion-ion Madelung potential.

In terms of 1e- eigenfunction $\psi_i(\vec{r})$ and occupancy n_i

$$\rho(\vec{r}) = \sum n_i \left|\psi_i(\vec{r})\right|^2 ; E[\rho(r)] \equiv E[\{\psi_i\}, R]$$

$$E[\{\psi_i\}, \vec{R}] = -\sum_i^{occ} n_i \int d\vec{r}\,\psi_i^*(\vec{r})\nabla^2\psi_i(\vec{r}) + \int d\vec{r}\,\rho(\vec{r})V_{ext}(\vec{r}) + \iint d\vec{r}\,d\vec{r}\,' \frac{\rho(\vec{r})\rho(\vec{r}')}{\left|\vec{r}-\vec{r}'\right|}$$

$$+ E_{xc}[\rho(\vec{r})] + \sum_{R,R'} \frac{Z_R Z_{R'}}{\left|\vec{R}-\vec{R}'\right|}$$

Variational principle => Euler equation

$$\frac{\delta E[\rho]}{\delta\rho}\Big|_{\rho=\rho_0} = 0 \qquad \delta\left\{E_{tot}^{el}[\rho(r)] - \mu\int\rho(r)dr\right\} = 0$$

Exchange energy

The electrostatic energy of a system will depend on relative orientation of the spins: The difference in energy defines the exchange energy.

If two spins are antiparallel, the wavefunction of the two electrons is symmetric
If two spins are parallel, Pauli exclusion principle requires the orbital part of the wave-function be anti-symmetric.Iinterchange of co-ordinates the wave-function changes sign)

Resulting Kohn-Sham (KS) equation:

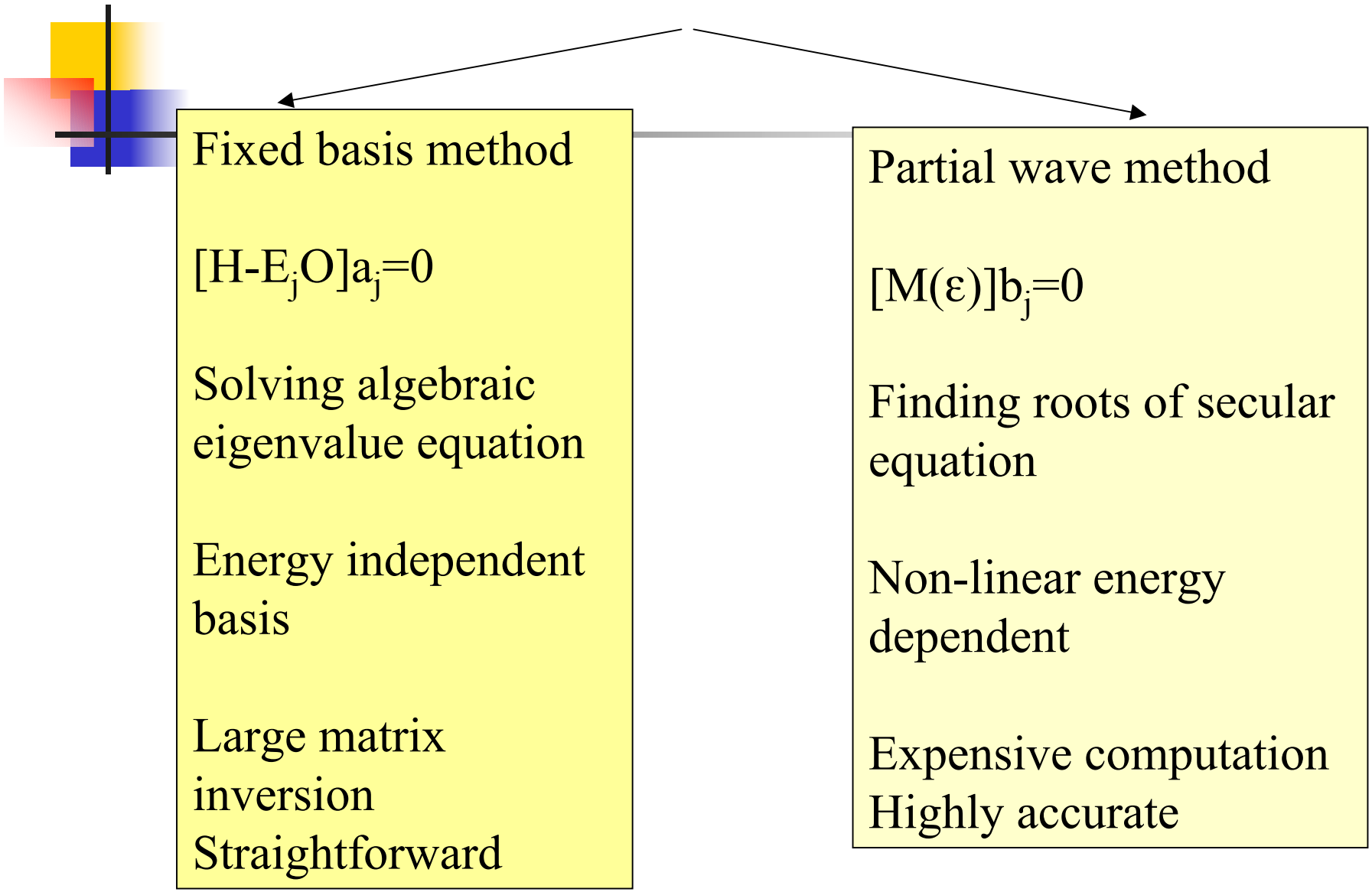
$$\{-\nabla^2 + V_{eff}(r)[\rho]\}\psi_i(r) = \varepsilon_i \psi_i(r)$$

where

$$V_{eff}(r)[\rho] = 2\int dr' \frac{\rho(r')}{|r - r'|} + V_{ext}(r) + \mu_{xc}(\rho(r))$$

V

Different $1e^-$ band structure methods



Muffin-tin orbital method: *Highly Efficient, Reasonably accurate*

Fixed (energy independent) basis function is derived from the energy dependent partial waves in the form of Muffin-tin Orbitals (MTO)

$$V_{MT}(r) = V_i(r_i) + \sum_R V_R(r_R) \equiv V_o + \sum_R v_R(r_R)$$

$$V_{MT}(r) = \begin{cases} v_R(r) - V_o \ldots\ldots\ldots r \le s_R \\ 0 \ldots\ldots\ldots\ldots\ldots\ldots r \ge s_R \end{cases}$$

Radial part

$$\left[-\frac{d^2}{dr^2} + \frac{l(l+1)}{r^2} + \nu_R(r) - E \right] r\chi_{Rl}(E,r) = 0$$

$$\left[-\frac{d^2}{dr^2} + \frac{l(l+1)}{r^2} - \kappa^2 \right] r y_l(\kappa r) = 0 \qquad \kappa = \sqrt{(E - V_o)}$$

Approximations:

Atomic sphere approximation (ASA)
Tight Binding approximations
Energy linearisation

- Neglect non-spherical parts of the potential
- Neglect interstitial region
- Neglect higher partial waves

$$\chi_{Rl}(E,r) = \phi_{Rl}(r) + (E - E_\nu)\dot{\phi}_{Rl}(r) + O(E - E_\nu)^2$$

$$\phi_{Rl} \equiv \phi(E_\nu, r) \qquad \text{and} \qquad \dot{\phi}_{Rl}(r) \equiv \partial\phi(E,r)/\partial E\,|_{E=E_\nu}$$

Using This Highly Efficient, Reasonably accurate method of calculation

Ψ_{total} the complete wave function of the crystal system
If Ψ_{Be2s} beryllium 2s state wave function.

The square of the overlap of Ψ_{total} with Ψ_{Be2s}, i.e $\left|\langle \psi_{total} | \psi_{Be2s} \rangle\right|^2$ represents the average number of 2s electrons in beryllium atom when it is implanted in a material.

LMTO Code

Inputs

- Lattice structure
- Partial co-ordinates of the constituents
- Atomic number
- Position of the implanted atom

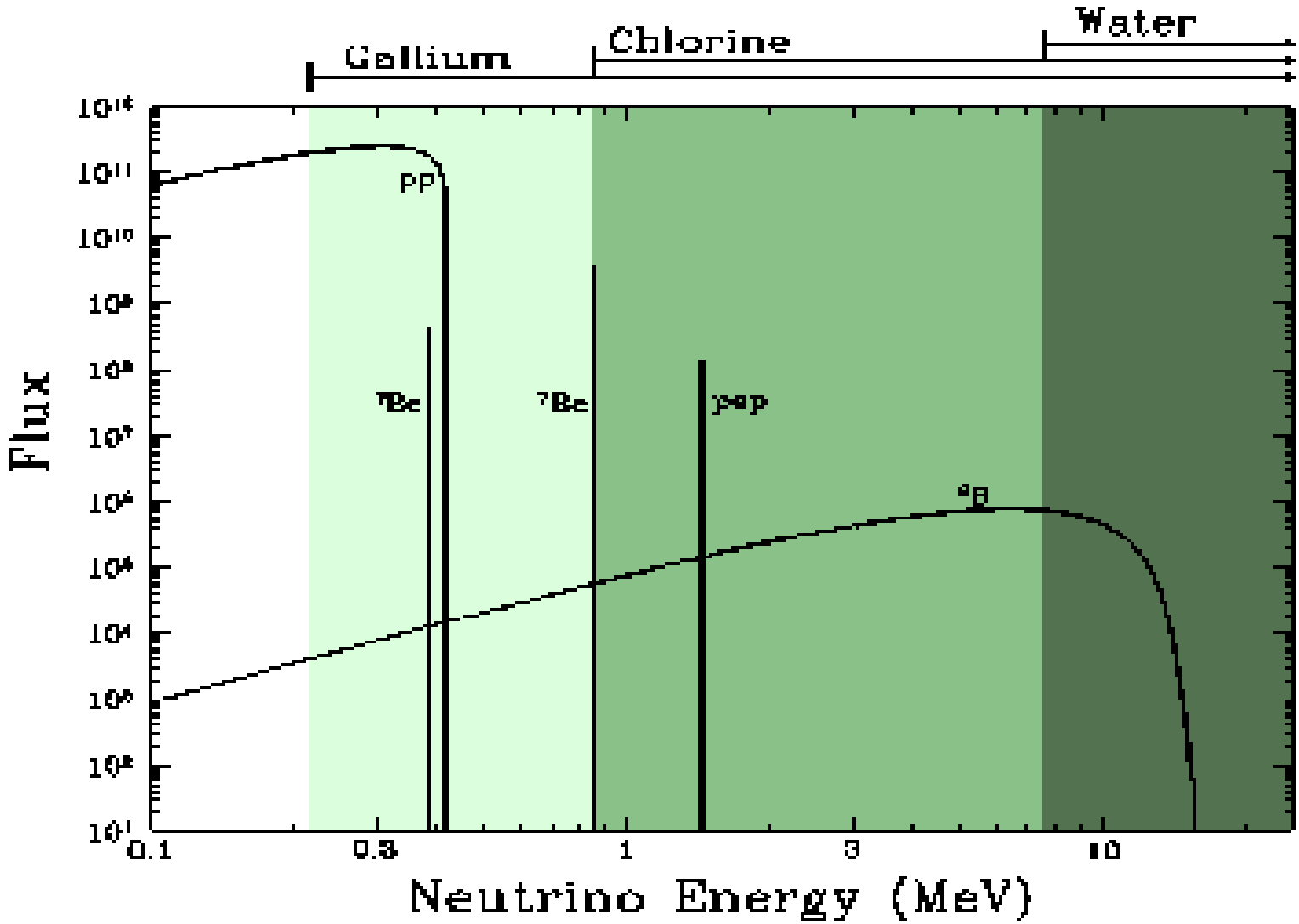
Steps:

Hartree potential calculation

Check for atomic sphere *Overlap*

Self-consistent calculation

Solar Neutrino Spectrum



Sudbury Neutrino Observatory (SNO)

http://www.sno.phy.queensu.ca